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A GRATING LOBE SUPPRESSION TECHNIQUE FOR  
LINEAR FREQUENCY-STEPPED PULSE TRAINS

JULY 1968

R.I. Millar

Prepared for

SURVEILLANCE AND WARNING SYSTEMS PROJECT OFFICE  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
L. G. Hanscom Field, Bedford, Massachusetts



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Project 6040  
Prepared by  
THE MITRE CORPORATION  
Bedford, Massachusetts  
Contract AF19(628)-5165

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## FOREWORD

This technical report "A Grating Lobe Suppression Technique for Linear Frequency-Stepped Pulse Trains" has been prepared under Technical Objective and Plans project 6040 Space Object Surveillance by The MITRE Corporation under Air Force Contract No. AF 19(628)5165. The cognizant Electronic Systems Division was the Strategic Forces Systems Planning Division of the Directorate of Planning and Technology.

## REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

SAMUEL S. HUMPHREY  
Lt Colonel, USAF  
Surveillance and Warning Systems Project Officer

## ABSTRACT

Linear frequency-stepped pulse trains, periodic in both time and frequency, have ambiguity functions which are periodic in both delay and doppler. One way of suppressing these ambiguities or grating lobes is time and frequency staggering of sub-pulses in the train, smearing out the grating lobes into a delay/doppler residue level. However this works well only with a large number of sub-pulses and a small pulse train duty factor. Another approach, described here, is to chirp code each sub-pulse, filling in the pulse train signal in the frequency domain. This suppresses ambiguities in a strip parallel to the delay axis, giving an unambiguous time response with some doppler tolerance. The technique appears to produce acceptable time responses with either a small or large number of sub-pulses and with various pulse train duty factors. The doppler tolerance, however, depends strongly upon the duty factor.

#### ACKNOWLEDGEMENT

The author is indebted to Mrs. Lorraine Moon for obtaining machine plots of the computed ambiguity functions, and to I. R. Smith for his criticism and review.

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## SECTION I

### INTRODUCTION

Synthesis of radar signals with large time-bandwidth products is conveniently accomplished with linear frequency-stepped pulse trains. Since both the pulse train time duration and the total frequency band covered are proportional to the number of pulses,  $N$ , the time-bandwidth product of the pulse train is proportional to  $N^2$ . The discrete nature of the signal makes it amenable to "digital" generation and processing techniques-- in particular, the transmitted frequencies can be locked to a set of stable reference oscillators, which can also be used as heterodyning frequencies in a crosscorrelation receiver. There are some practical advantages in using frequency-stepped pulse train signals with a wideband SOI radar.

These advantages are offset by the range and doppler ambiguities of the signal which arise because the waveform is periodic in both time and frequency, and greatly restrict the use of this signal in multiple target environments. One way in which these ambiguities can be partly suppressed is by staggering of the pulse position and/or frequency, smearing out the ambiguities. However, it is difficult to obtain low range and doppler residues

by this method, which is analogous to non-uniform spacing of elements in an array antenna, unless a very large number of pulses are available. Another way, which is proposed and discussed in this paper, is to code each sub-pulse with a chirp signal, whose time-bandwidth product is much smaller than the overall pulse train time-bandwidth product, "filling in" the frequency spectrum of the pulse train. In this way, range ambiguities can be suppressed in a limited strip of the range/doppler plane, parallel to the range axis. A target space which is extended in range, but bounded in doppler, typical of many radar applications, can be accommodated with such an ambiguity surface.

## SECTION II

### SIGNAL DESIGN CONSIDERATIONS

The time response of a matched filter to a doppler shifted signal is given by Woodward's ambiguity function.\* The ambiguity function of a linear frequency-stepped pulse train has been calculated by Rihaczek [1] as

$$|X(\tau, \nu)|^2 = \frac{1}{N^2} \sum_{m=-(N-1)}^{N-1} |X_p(\tau - mT, \nu - mF)|^2 \quad (1)$$

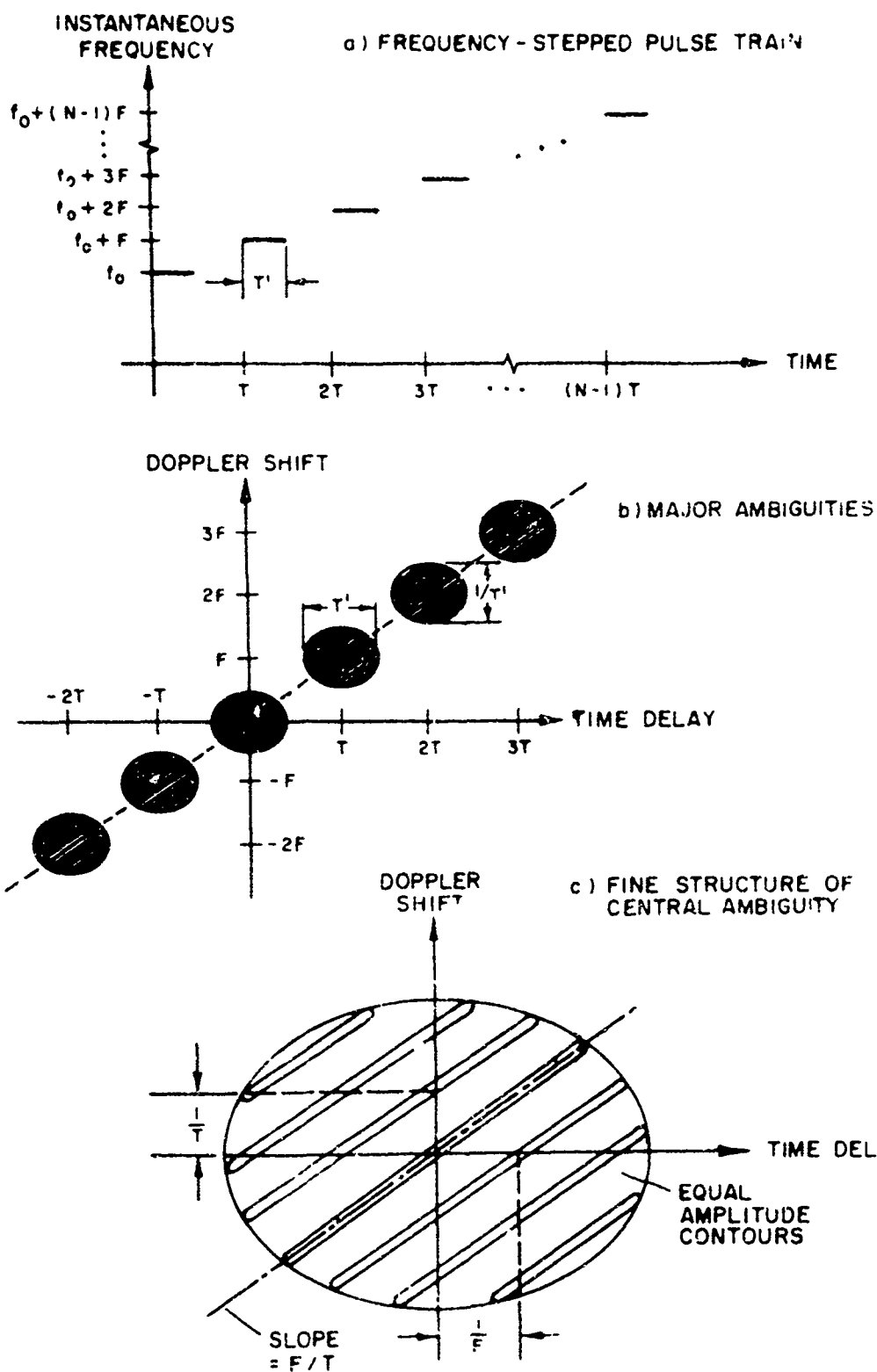
$$\frac{\sin^2 \pi [N - |m|] [T\nu - F(\tau - mT)]}{\sin^2 \pi [T\nu - F(\tau - mT)]}$$

where  $N$  is the number of pulses in the train;  $T$  is the pulse repetition period;  $F$  is the frequency step between pulses; and  $|X_p(\tau, \nu)|^2$  is the ambiguity function of a single pulse.

The signal waveform is illustrated in figure 1(a). A burst or train of  $N$  coherent sub-pulses is transmitted sequentially. After the entire burst is over, the receiver is

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\* Restrictions on the use of the ambiguity function for very large time-bandwidth products, as well as the physical interpretation of the results in terms of the receiver parameters, are discussed in Section IV.



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Figure 1. AMBIGUITY FUNCTION OF FREQUENCY-STEPPED PULSE TRAIN

turned on to receive the echo of the entire pulse burst, and stays on until the next burst. No attempt is made to receive in the time intervals between sub-pulses, so that the minimum range is determined by the pulse burst time duration,  $NT$ . The ambiguity function, with  $2N-1$  major ambiguities, is sketched in figure 1(b). In most applications, the frequency step,  $F$ , is much larger than the doppler spread of the target space, so that only the central ambiguity ( $m=0$ ) is of importance. The fine structure of this central ambiguity is sketched in figure 1(c). One sees that the delay/doppler ridge ambiguities will allow interference between target returns, unless all targets are confined to a spread of less than  $1/F$  in delay and less than  $1/T$  in doppler. Since the former condition is often not achieved in practical situations, some way must be found to suppress at least the time delay ambiguities.

With monotone sub-pulses, the ambiguities, or grating lobes, along the time delay axis can be suppressed by making the sub-pulse time duration,  $T'$ , less than the grating lobe spacing,  $1/F$ , as shown in figure 2(a). This condition is derived from the ambiguity function of a single monotone pulse, given by Cook and Bernfeld [2], as

b.) DELAY / DOPPLER  
CONTOUR MAP

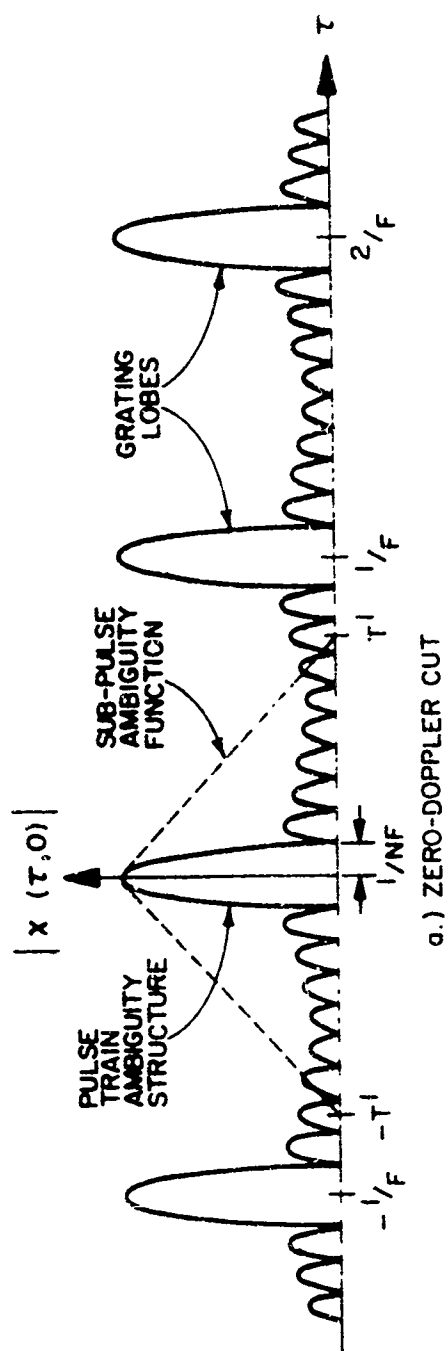
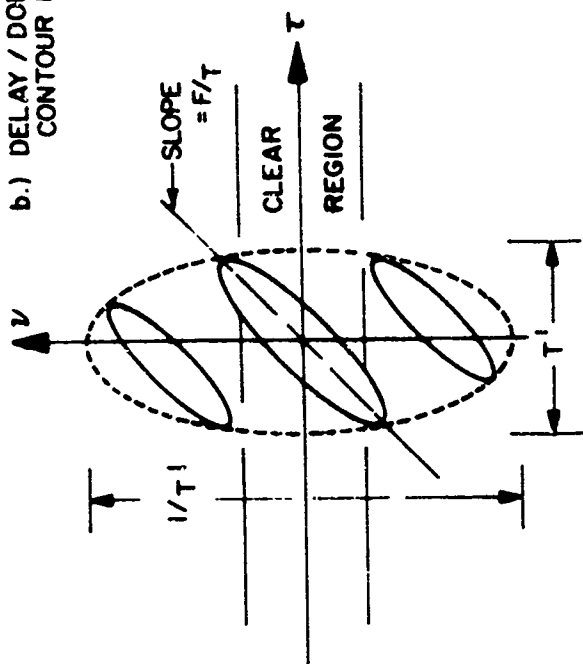


FIGURE 2. AMBIGUITY FUNCTION WITH SHORT MONOTONE SUB-PULSES

$$|x_p(\tau, \nu)|^2 = \left(1 - \frac{|\tau|}{T'}\right)^2 \left| \frac{\sin \pi \nu T' (1 - |\tau|/T')}{\pi \nu T' (1 - |\tau|/T')} \right|^2, |\tau| < T'$$

$$= 0, |\tau| \geq T' \quad (2)$$

The first grating lobe along the doppler axis cannot be suppressed, however, since the first doppler null in the monotone pulse ambiguity function occurs at  $\nu = 1/T'$ , and one cannot set  $1/T'$  equal to  $1/T$  and still have a pulse train signal. Thus, an unambiguous time response can only be achieved for a limited span of doppler shifts, encompassing the "clear" region shown in figure 2(b).

The attainable time-bandwidth product of the pulse train is limited under these conditions. If we define  $\delta = T'/T$  as the duty factor of the pulse train,<sup>\*</sup> then the overall time-bandwidth product,  $(\beta\tau)_{\text{overall}}$ , is given by

$$(\beta\tau)_{\text{overall}} \cong N^2 FT = \frac{N^2 FT'}{\delta} \leq \frac{N^2}{\delta}, N \gg 1 \quad (3)$$

---

\* The overall radar duty factor is much less than  $\delta$ , since time must be allowed to receive signals from each pulse train or burst. Thus values of  $\delta$  approaching unity are possible without requiring simultaneous transmission and reception.

Thus, large  $(\beta\tau)_{\text{overall}}$  can only be achieved with a large number of pulses or with a small duty factor, which limits the transmitted pulse energy.

The waveform illustrated in figure 3(a) offers more flexibility to the radar designer. Linear frequency modulation is imposed on each sub-pulse in the train, giving a pulse train of "chirp" sub-pulses, with starting frequencies stepped by  $F$  cycles per second. The ambiguity function of a single chirp pulse is also given in reference [3] as

$$|x_p(\tau, \nu)|^2 = \left(1 - \frac{|\tau|}{T'}\right)^2 \left| \frac{\sin(b\tau - \pi\nu)(T' - |\tau|)}{(b\tau - \pi\nu)(T' - |\tau|)} \right|^2, |\tau| < T'$$

$$= 0, |\tau| \geq T' \quad (4)$$

where  $b/\pi$  is the slope of the linear FM in cycle/sec.<sup>2</sup> We now inquire into how this slope should be chosen to provide suppression of the grating lobes along the delay axis.

The oscillatory term in equation (1) reduces to  $\sin^2 \pi NF\tau / \sin^2 \pi F\tau$ , for points along the delay axis, where  $m=0$  and  $\nu=0$ , so that the grating lobes occur at equally spaced intervals of  $1/F$  along the delay axis. If the zeroes of the chirp sub-pulse could be made to occur at the same points as



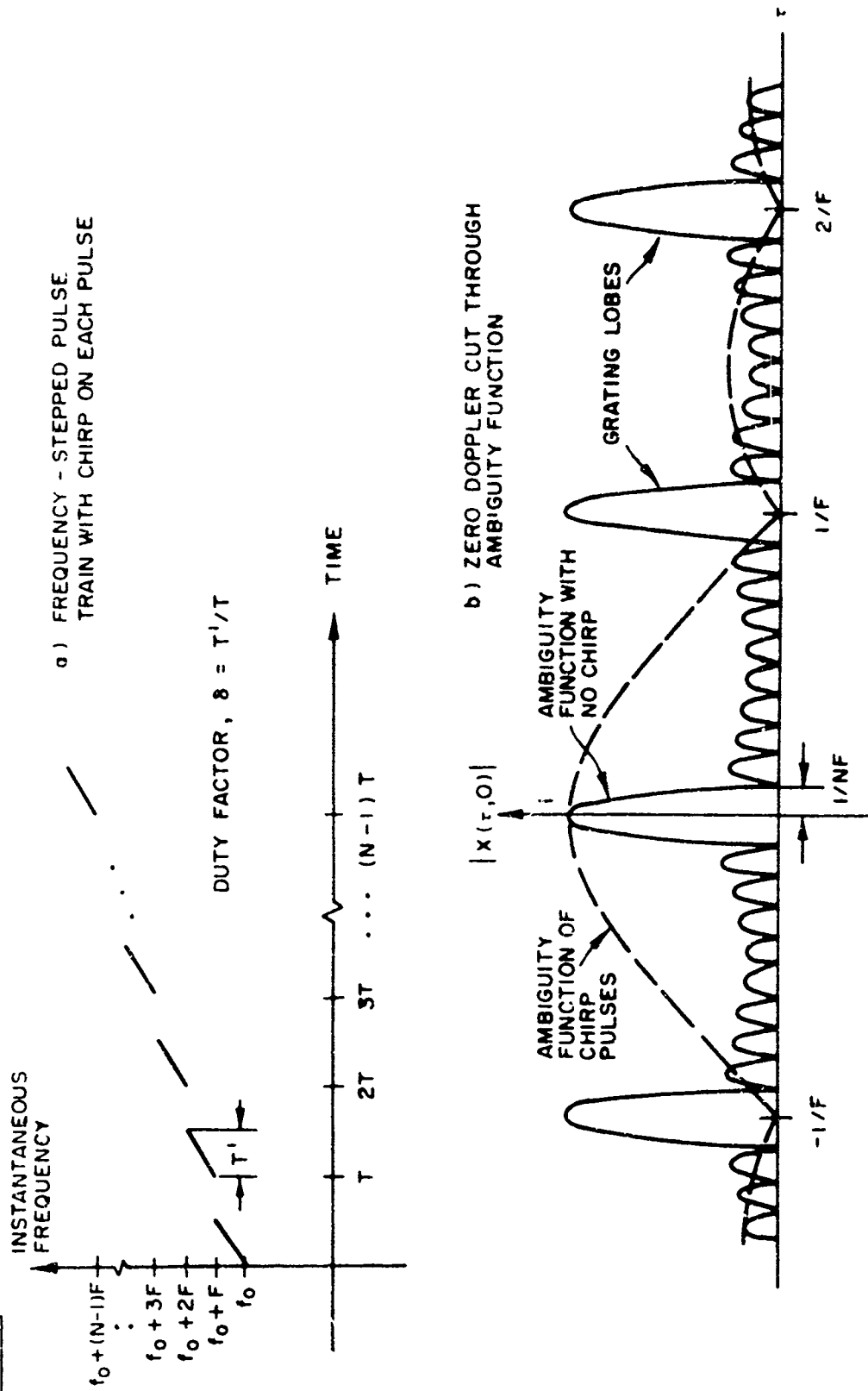


Figure 3. SUPPRESSION OF RANGE AMBIGUITIES

shown in figure 3(b), maximum suppression of the grating lobes would occur. This would require that

$$|x_p(\tau, 0)|^2 = \left(1 - \frac{|\tau|}{T'}\right)^2 \left| \frac{\sin b\tau(T' - |\tau|)}{b\tau(T' - |\tau|)} \right|^2 \bigg|_{\tau = \frac{n}{F}} = 0 \quad (5)$$

which in turn requires that

$$\sin b\tau(T' - |\tau|) \bigg|_{\tau = \frac{n}{F}} = 0 \quad (6)$$

But this cannot be achieved, in general. However, if the chirp pulse has a moderately large time-bandwidth product, the first few nulls can meet this condition. That is, if

$|\tau| = |n|/F \ll T'$ , which implies that  $|n| \ll FT'$ , where  $FT'$  is the time-bandwidth product of the chirp sub-pulse, then

$$\sin b\tau(T' - |\tau|) \bigg|_{\tau = \frac{n}{F}} \cong \sin b\tau T' \bigg|_{\tau = \frac{n}{F}} = \sin \frac{nb T'}{F} = 0 \quad (7)$$

which immediately gives the result that

$$\frac{b}{\pi} = \frac{F}{T'} \quad (8)$$

in order to make the first few nulls of the chirp sub-pulse coincide with the grating lobes. Physically, this means that the chirp sub-pulse should have a slope such that it covers the entire interpulse frequency step in the sub-pulse time duration, giving contiguous coverage of the frequency band, as illustrated in figure 3(a). The constraint,  $|n| \ll FT'$ , means that the number of chirp sub-pulse nulls which coincide with grating lobes increases as the sub-pulse time-bandwidth product,  $FT'$ , increases. Eventually, as  $|n|$  increases, some grating lobes will coincide with sidelobes of the chirp sub-pulse, but if  $|n|$  is large, the amplitude of the sidelobes will be small enough to provide suppression of the grating lobes.

Away from the delay (zero-doppler) axis, some of the grating lobes begin to reappear, as shown in figure 4. This is due to the difference in slopes between the delay/doppler grating lobe ridges and the delay/doppler ridge of the chirp sub-pulse. Because of this difference in slopes, the nulls of the chirp signal response can coincide with the grating lobes of the pulse train response only at or near some particular value of doppler

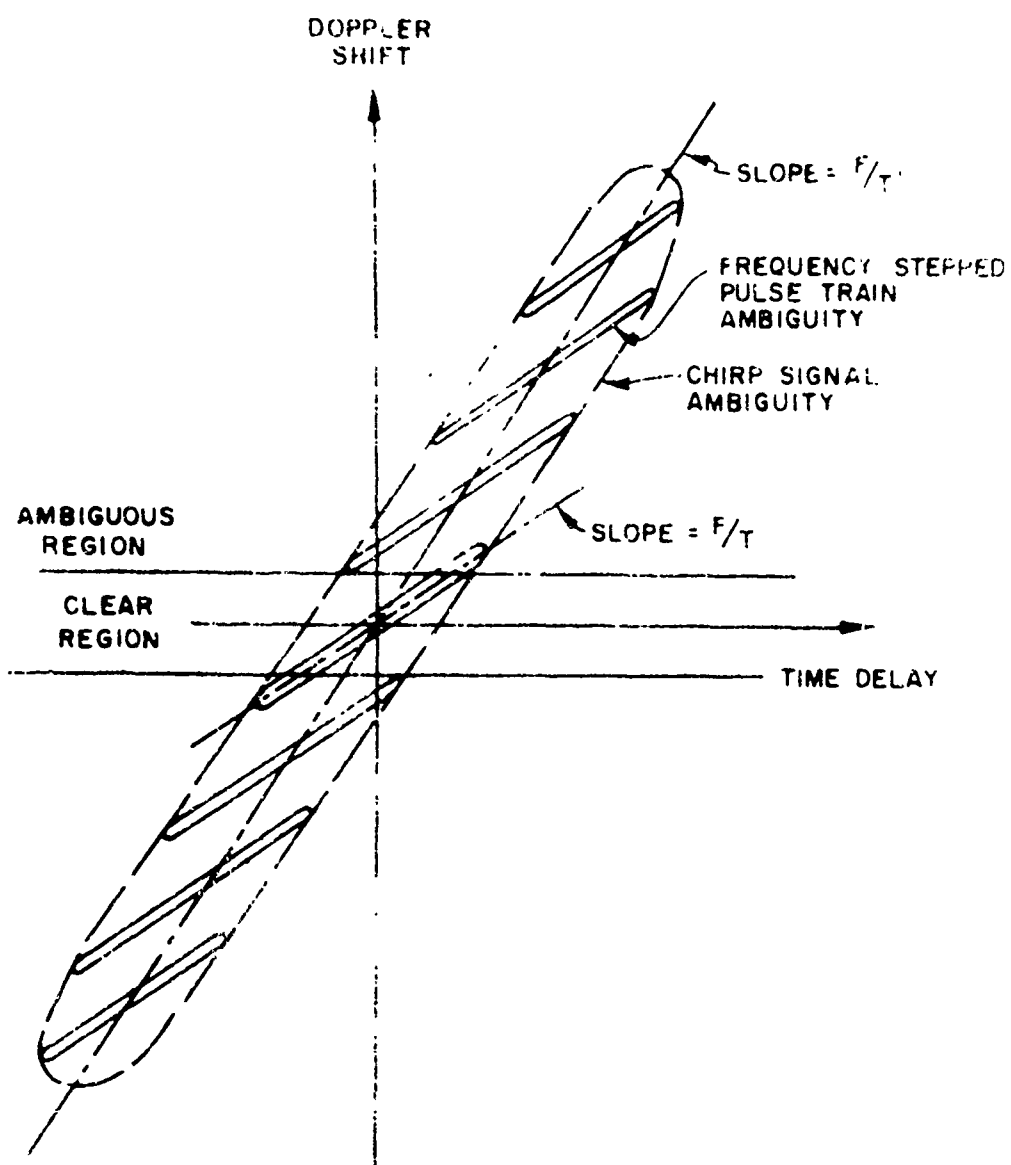


Figure 4. CLEAR REGION NEAR TIME DELAY AXIS

shift. Thus, an unambiguous range response (the "clear" region of figure 4) is obtained only in a strip of the delay/doppler plane parallel to the delay axis, as was the case for the monotone sub-pulse. The width of the strip depends upon the pulse train duty factor,  $\delta = T'/T$ . If  $\delta$  is near unity, the two slopes in question are nearly the same and the clear region is fairly wide in doppler. If  $\delta$  is small, on the other hand, the two sets of delay/doppler ridges rapidly diverge and the clear region is very narrow in doppler.

The advantage of using the chirp sub-pulse as opposed to a monotone sub-pulse lies in the larger time-bandwidth products it makes available. We can write an equation analogous to equation (3) for the chirp case as

$$(\beta\tau)_{\text{overall}} \cong N^2 FT' = \frac{N^2 FT'}{\delta} = \frac{N^2}{\delta} (\beta\tau)_{\text{sub-pulse}} \quad (9)$$

Thus, we have an extra degree of freedom in designing the radar signal. A very large time-bandwidth product might be synthesized with a small number of sub-pulses, each having a large time-bandwidth product; with a large number of sub-pulses, each having a moderate time-bandwidth product; or various cases in between. In addition, it is practical to operate

with large pulse train duty factors, obtaining a large signal energy with limited peak power.

### SECTION III

#### COMPUTED AMBIGUITY FUNCTIONS

To make the preceding discussion more concrete, a number of representative ambiguity functions have been calculated on the IBM 7030 digital computer and plotted with the Benson-Lehner plotter. These plots have been normalized to make clear the fundamental parameters involved. By appropriate manipulations of equations (1), (4), and (8), the four parameters  $F$ ,  $T$ ,  $T'$ , and  $N$  may be reduced to the three more fundamental parameters,  $(\beta\tau)_{\text{overall}}$ ,  $N$ , and  $\delta$ , used in the following equations:

$$|x(\tau', \nu')|_{m=0}^2 = \frac{1}{N^2} |x_p(\tau', \nu')|^2 \frac{\sin^2 \pi N [\nu' - \tau']}{\sin^2 \pi [\nu' - \tau']} \quad (10)$$

$$|x_p(\tau', \nu')|^2 = \left(1 - \frac{|\tau'|}{K}\right)^2 \left| \frac{\sin \pi (\tau' - \delta \nu') \left(1 - \frac{|\tau'|}{K}\right)}{\pi (\tau' - \delta \nu') \left(1 - \frac{|\tau'|}{K}\right)} \right|^2, |\tau'| < K$$

$$= 0, |\tau'| \geq K \quad (11)$$

$$K = (\beta\tau)_{\text{overall}} \cdot \frac{\delta}{N} = (\beta\tau)_{\text{sub-pulse}} \quad (12)$$

The new variables,  $\tau' = F\tau$  and  $\nu' = T\nu$ , are normalized so that the ridge ambiguities intersect both the delay and doppler axes at unit intervals.

All the cases presented assumed a pulse train time-band-width product,  $(\beta\tau)_{\text{overall}}$  of 100,000. The first group of cases considered a train of  $N=10$  pulses, with duty factors,  $\delta$ , of 0.1, 0.5, and 0.9, to obtain an idea of how the duty factor affects the doppler tolerance of the waveform. A second group of cases compares trains of 3, 10, and 30 pulses, all with a duty factor of 0.5, to see how the structure of the ambiguity function depends upon the number of pulses.

As a basis of comparison for the other cases, the ambiguity function for a 10 pulse, 50% duty factor train of monotone sub-pulses was computed from equation (1) and (2), after normalizing them in the same way as we did the equations for chirp coded sub-pulses. This is shown in figure 5. Only one cut at zero doppler is given, as all the doppler cuts are very nearly alike in shape, being merely translated in delay. One sees the characteristic periodic grating lobe structure,



resulting from the line spectrum of the pulse train.

Figure 6 is the ambiguity function of a 10 pulse train of chirp sub-pulses, with a 10% duty factor. Parts (a) through (e) are cuts at increments of 0.05 (5% of the distance to the first doppler ambiguity at  $1/T$ ) in  $\nu'$ , the normalized doppler shift, starting at zero. Since  $|x(\tau', \nu')|^2$  has symmetry about the origin, only cuts for positive doppler are shown. Figure 7 shows the ambiguity function for a 10 pulse train of chirp sub-pulses with a 50% duty factor, for the same cuts as in figure 6. We see that the larger duty factor gives better suppression of the grating lobes, for both zero and non-zero incremental doppler. The better suppression at zero-doppler is probably due to the larger time-bandwidth product of each sub-pulse, which makes its zeroes more nearly equally spaced. The better doppler tolerance is due to the more "filled in" character of the signal in the time domain. When the duty factor is increased to 90%, as shown in figure 8, the zero-doppler response improves only slightly, but the doppler tolerance becomes much better, again due to more filling in of the signal in the time domain.

One might inquire about how well grating lobes are suppressed for larger values of incremental delay than are plotted. Figure 6(a) appears to be a suspicious case in this

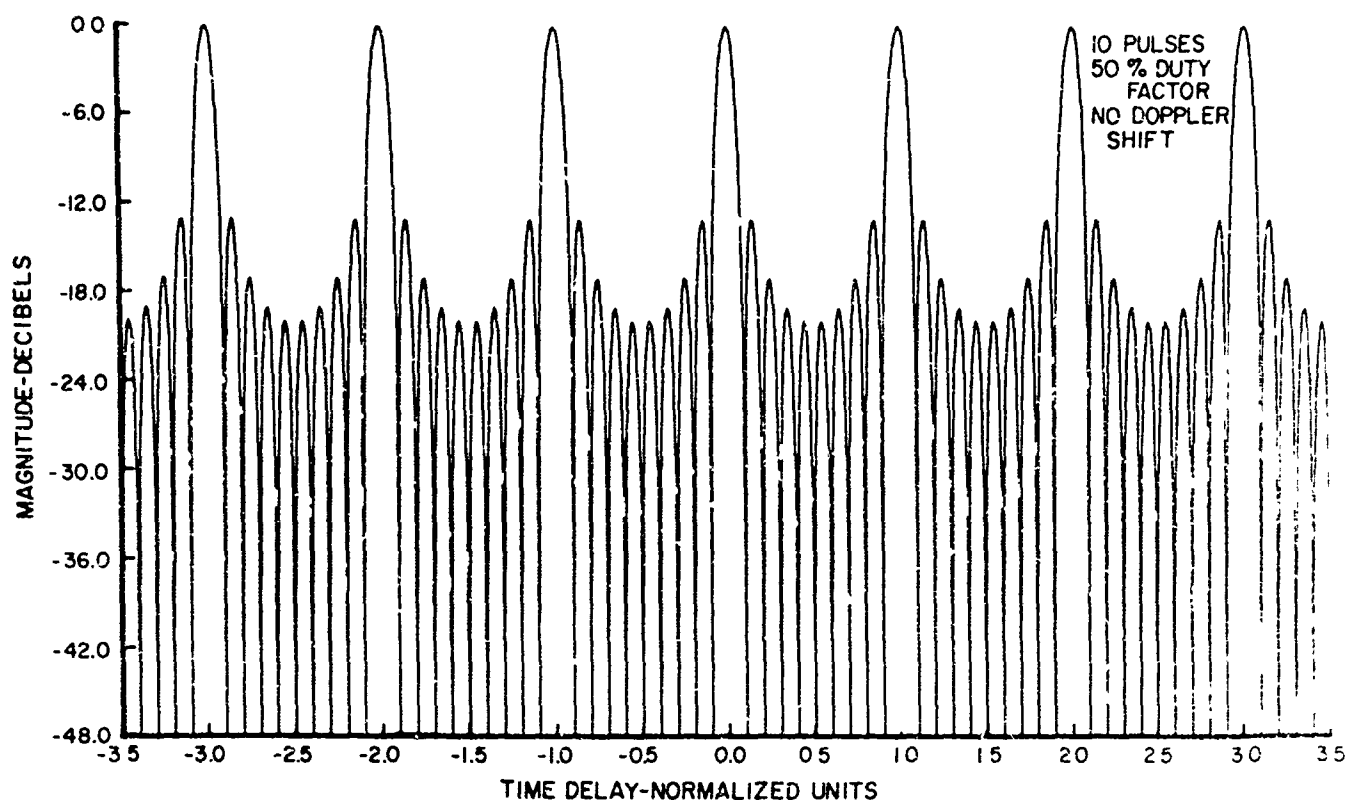


FIGURE 5. AMBIGUITY FUNCTION WITH MONOTONE SUB-PULSES

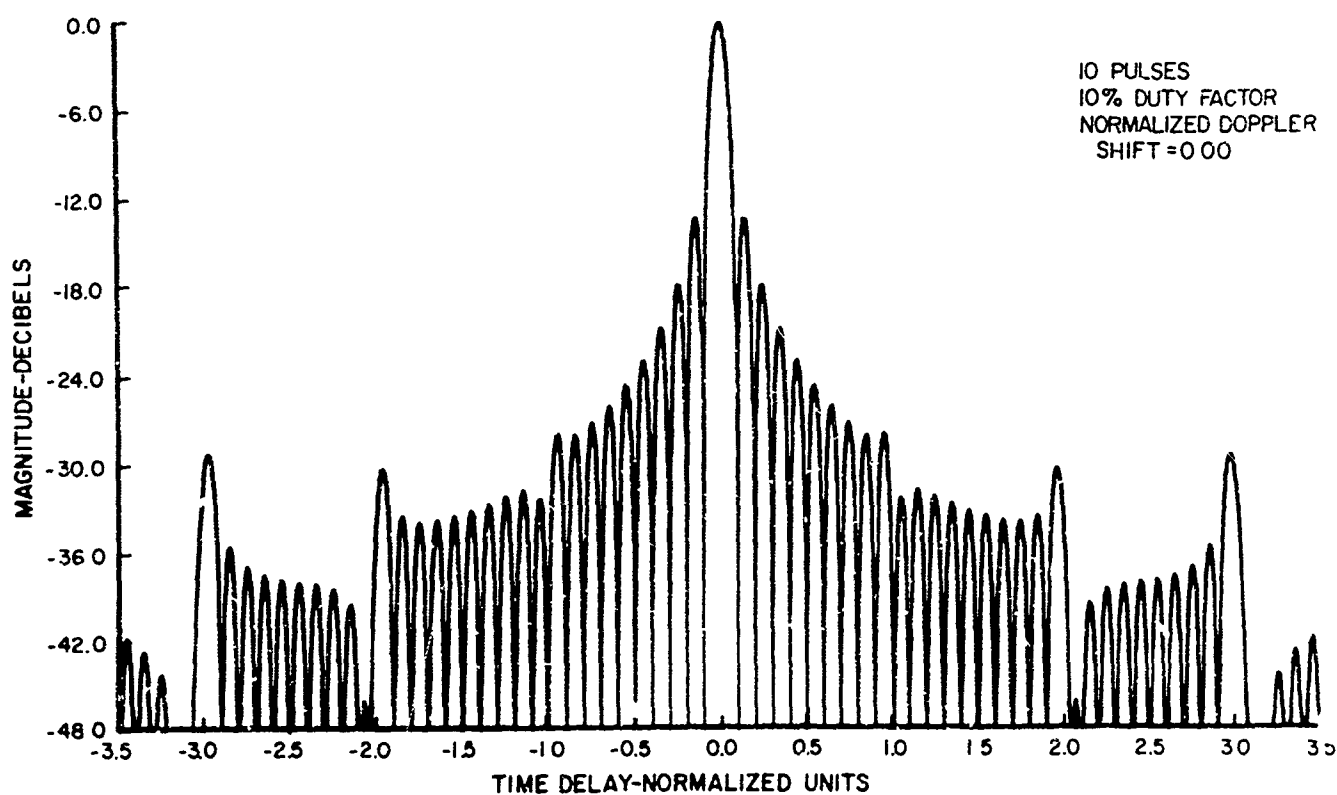


FIGURE 6(a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

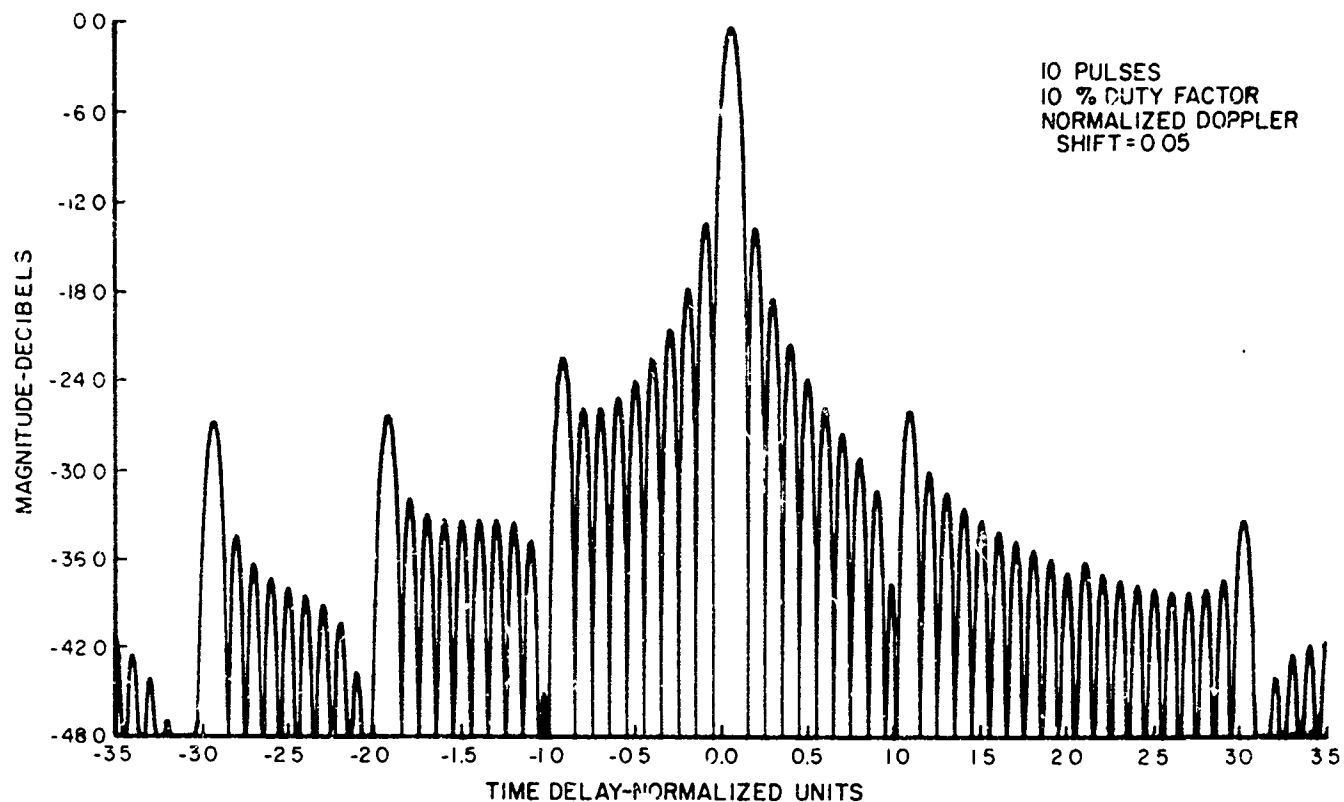


FIGURE 6 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

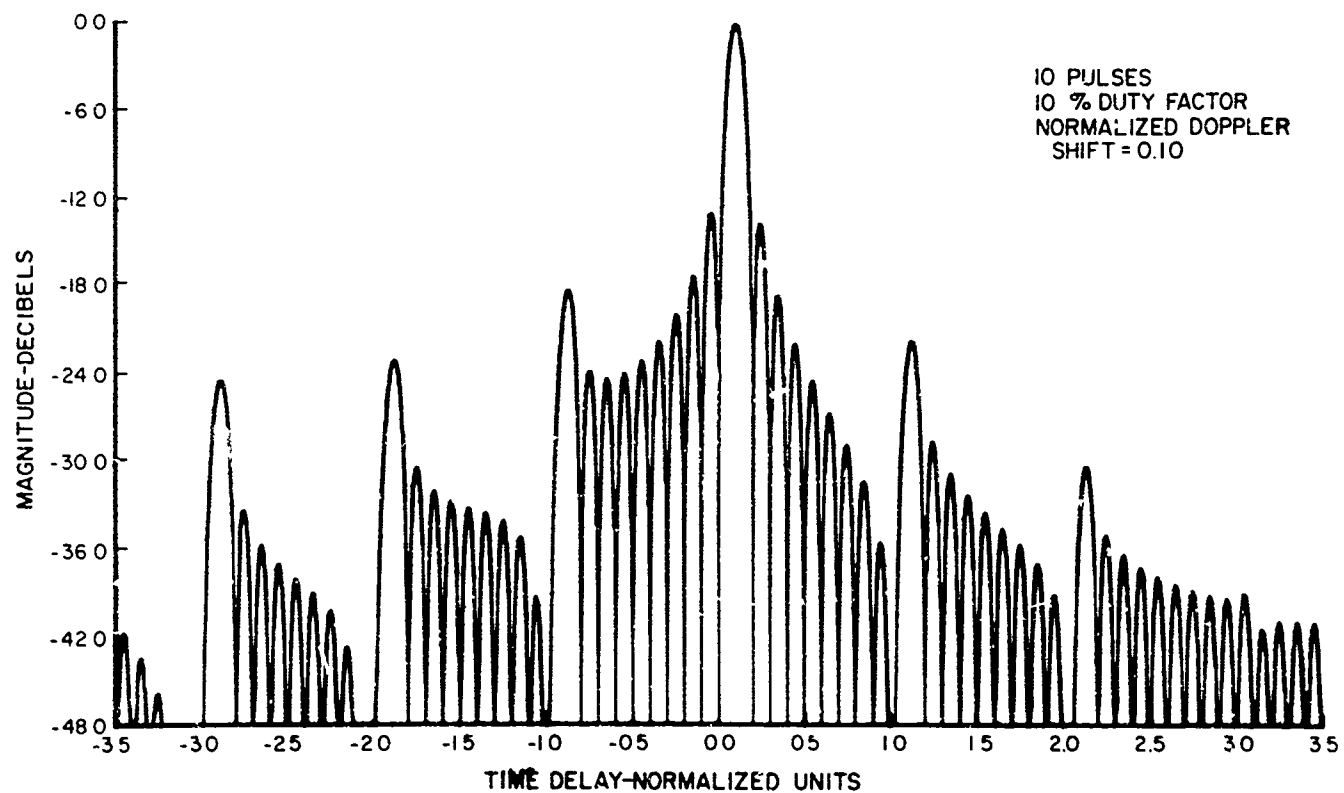


FIGURE 6 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

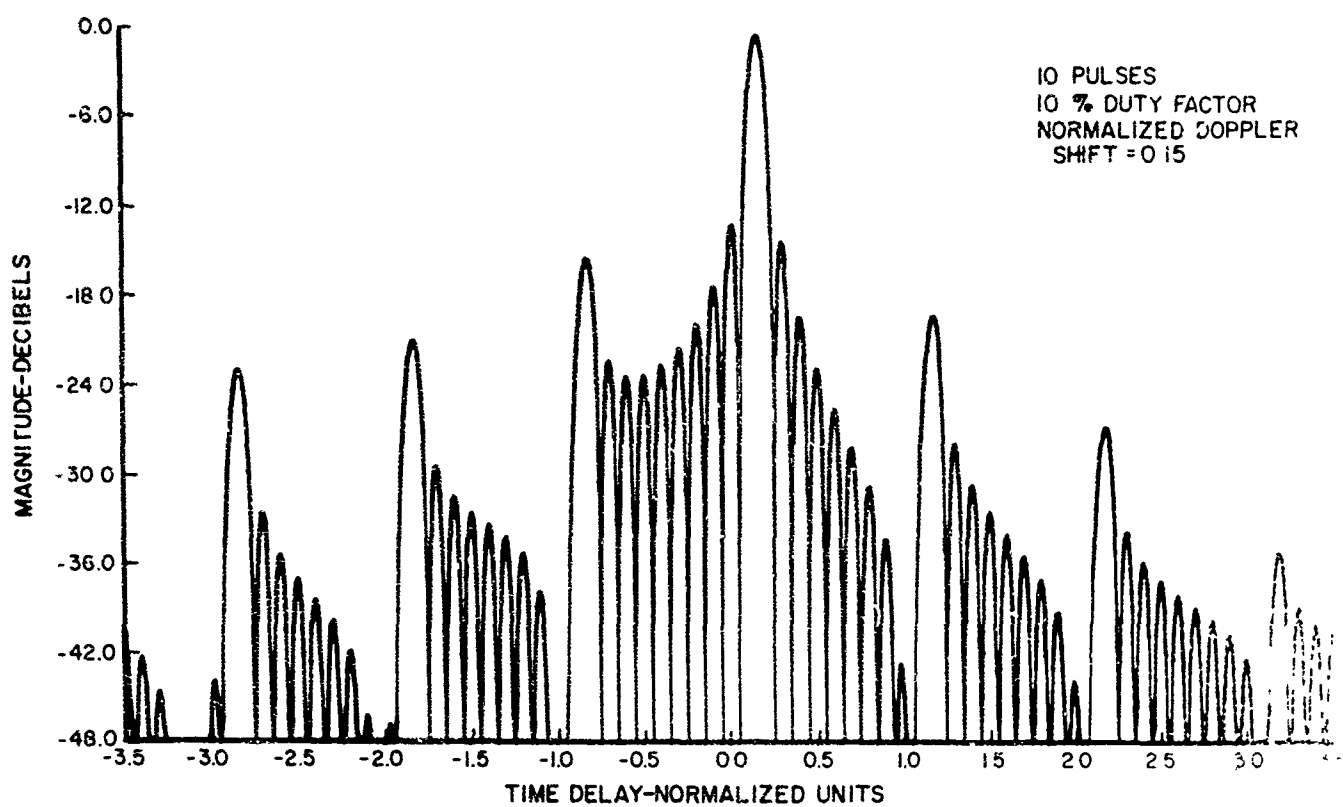


FIGURE 6(d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

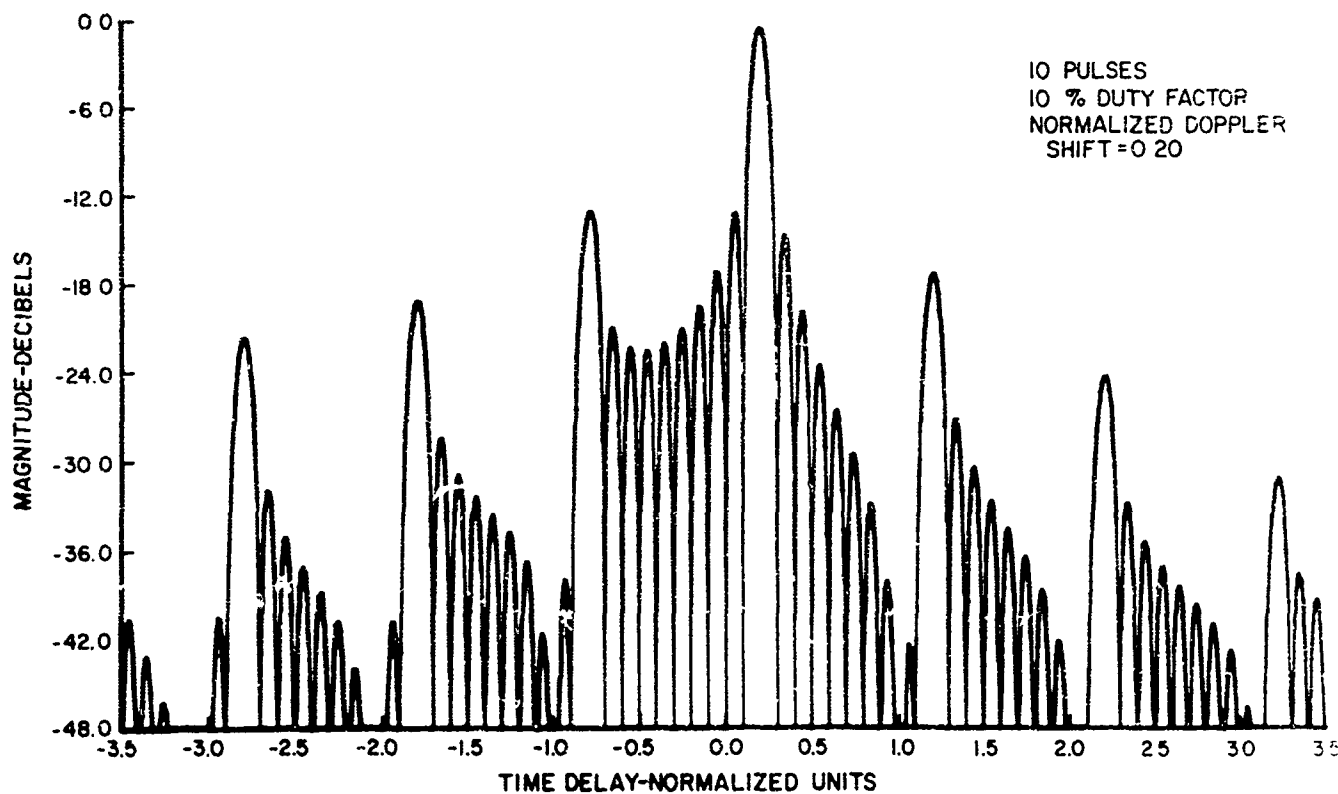


FIGURE 6(e.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

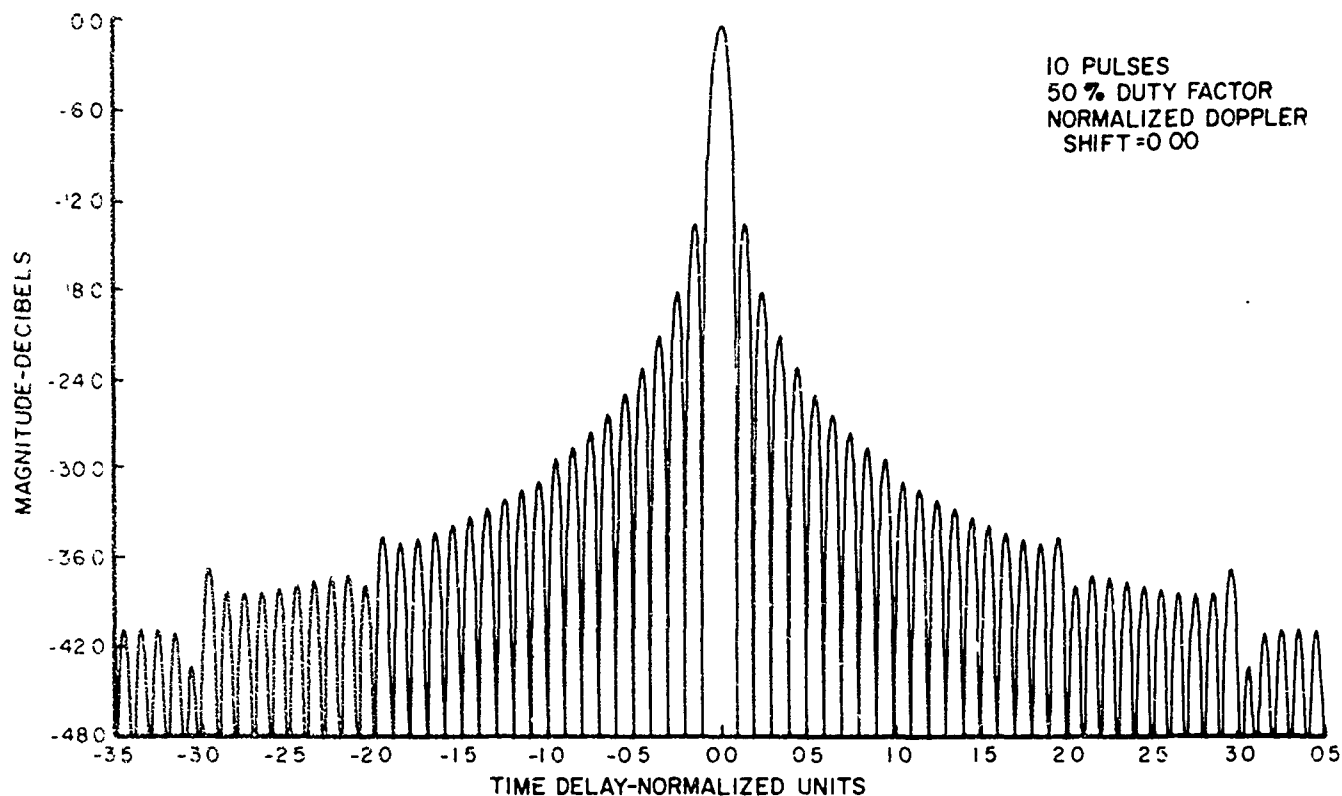


Figure 7 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

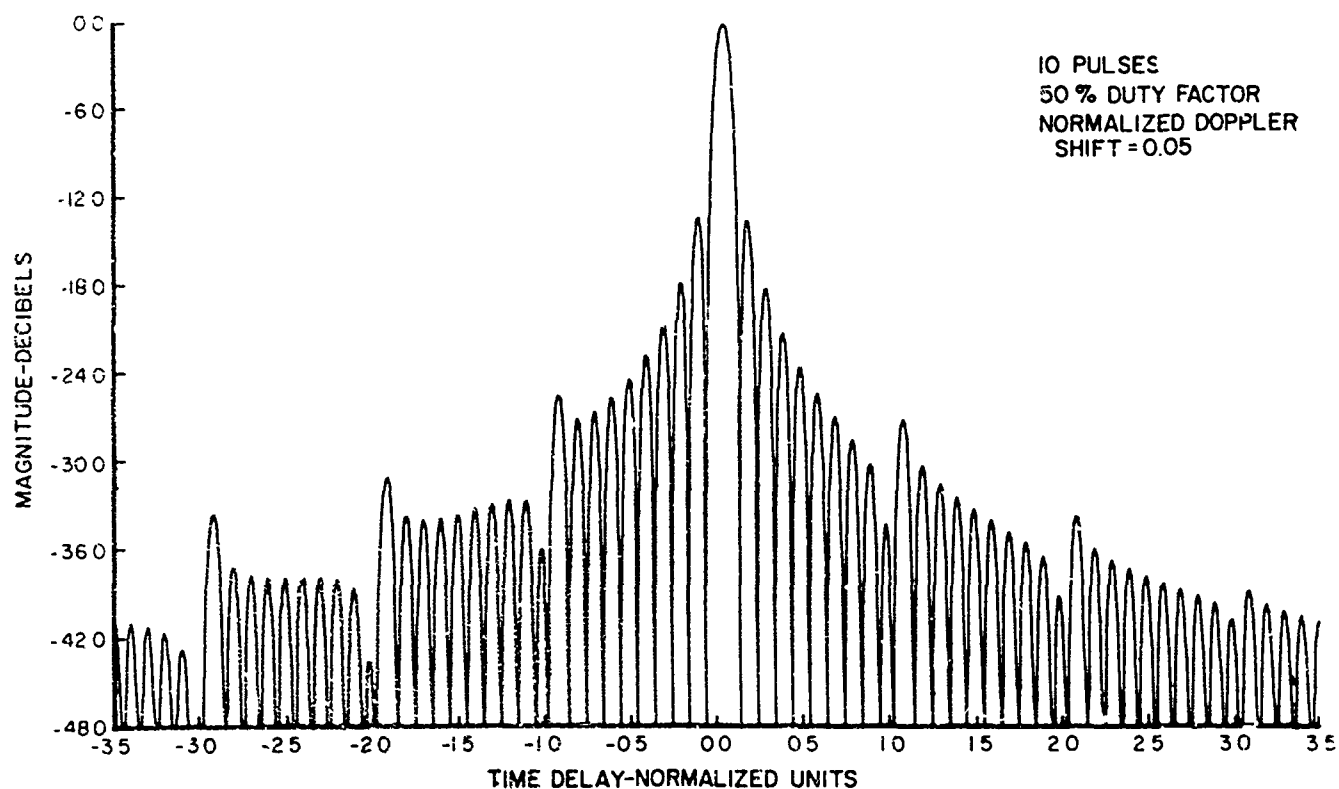


Figure 7 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

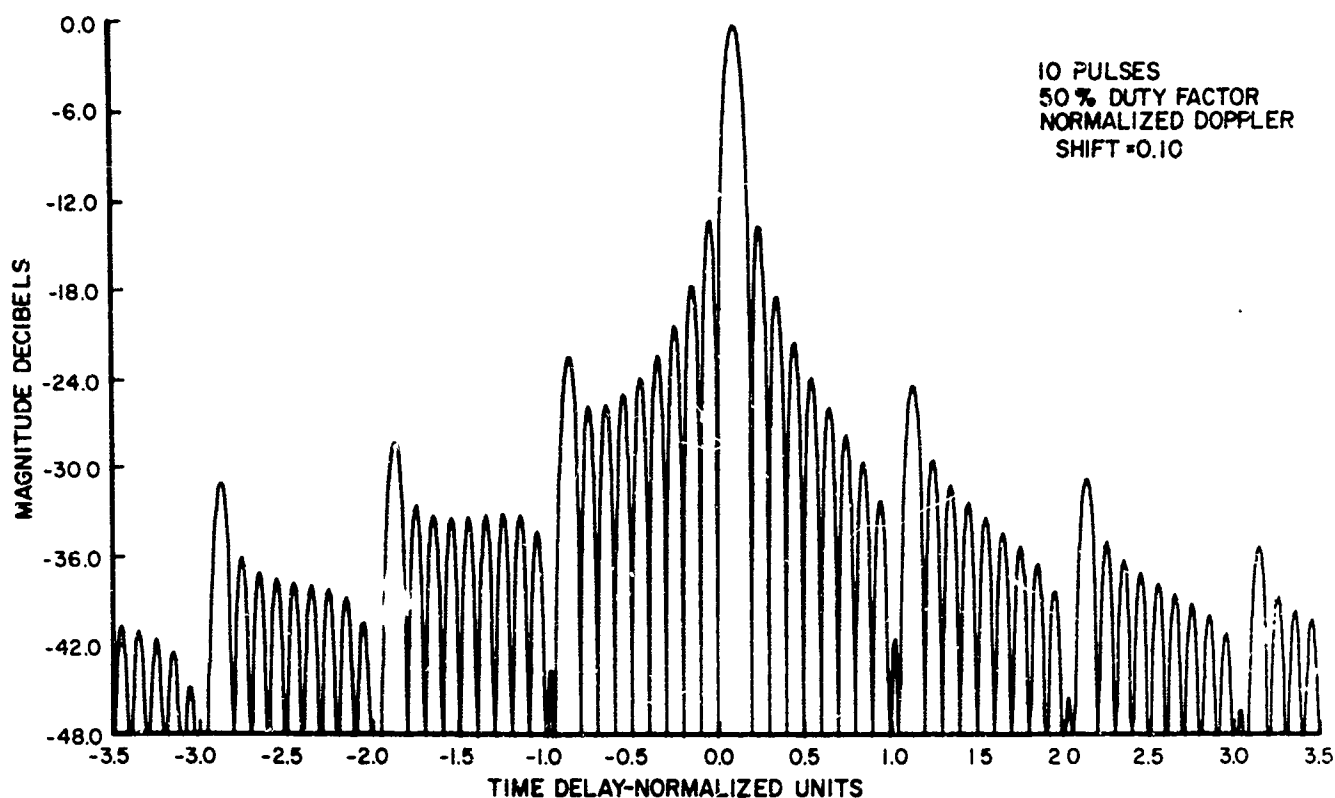


Figure 7 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

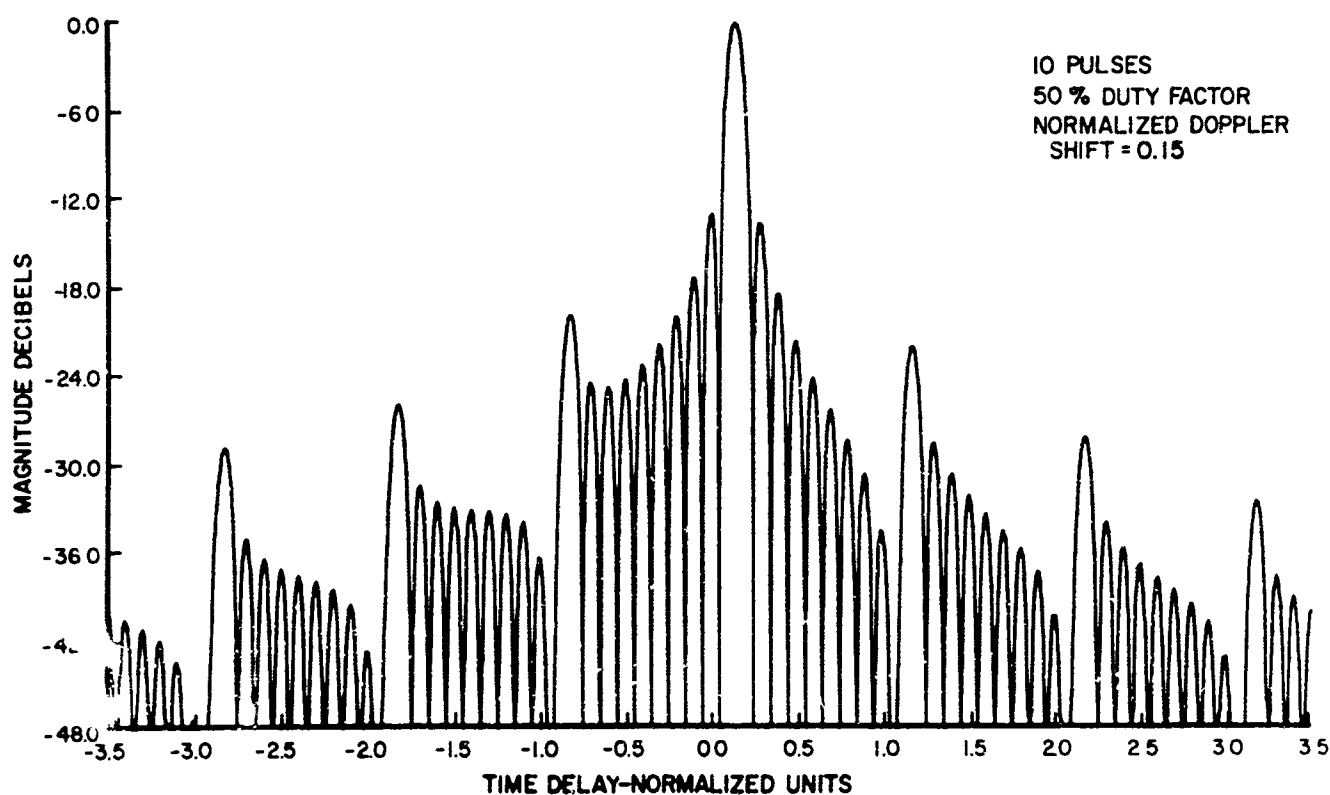


Figure 7 (d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

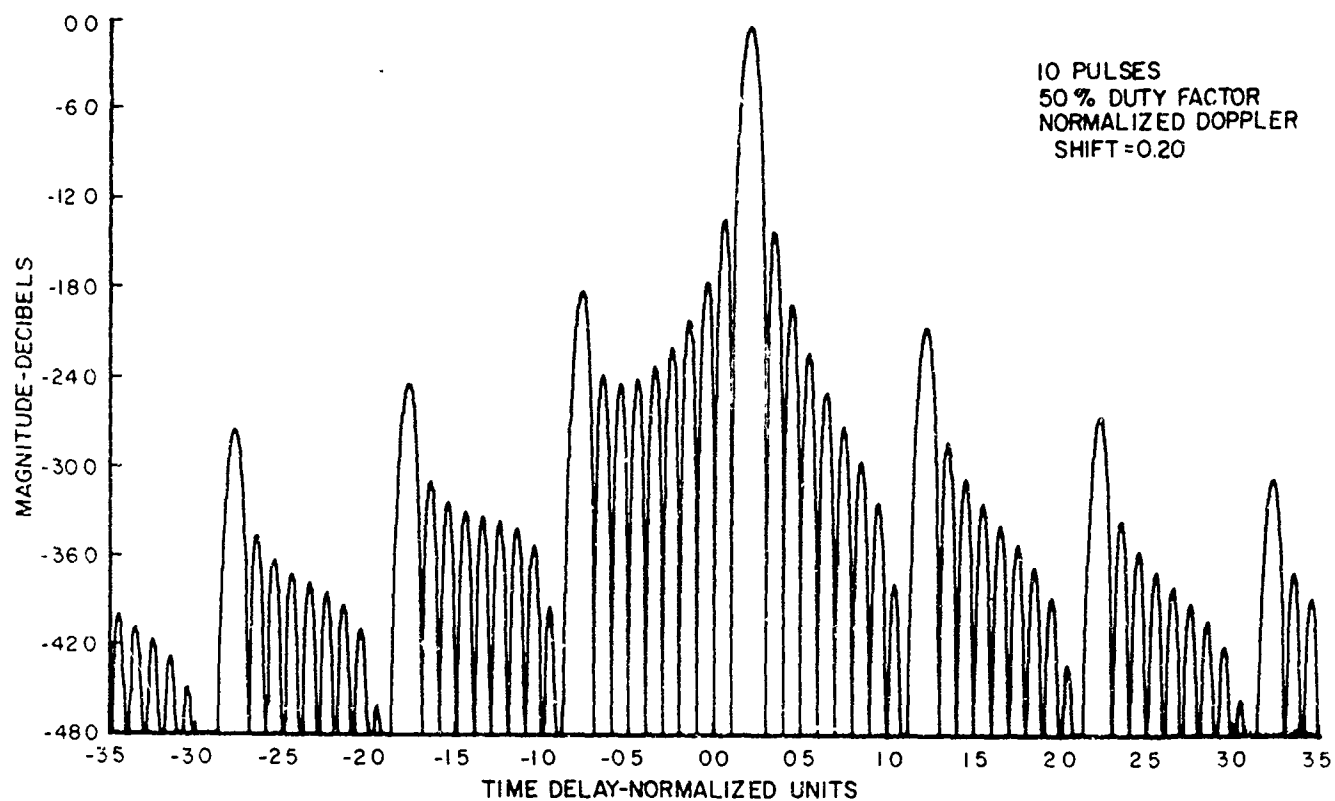


Figure 7 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

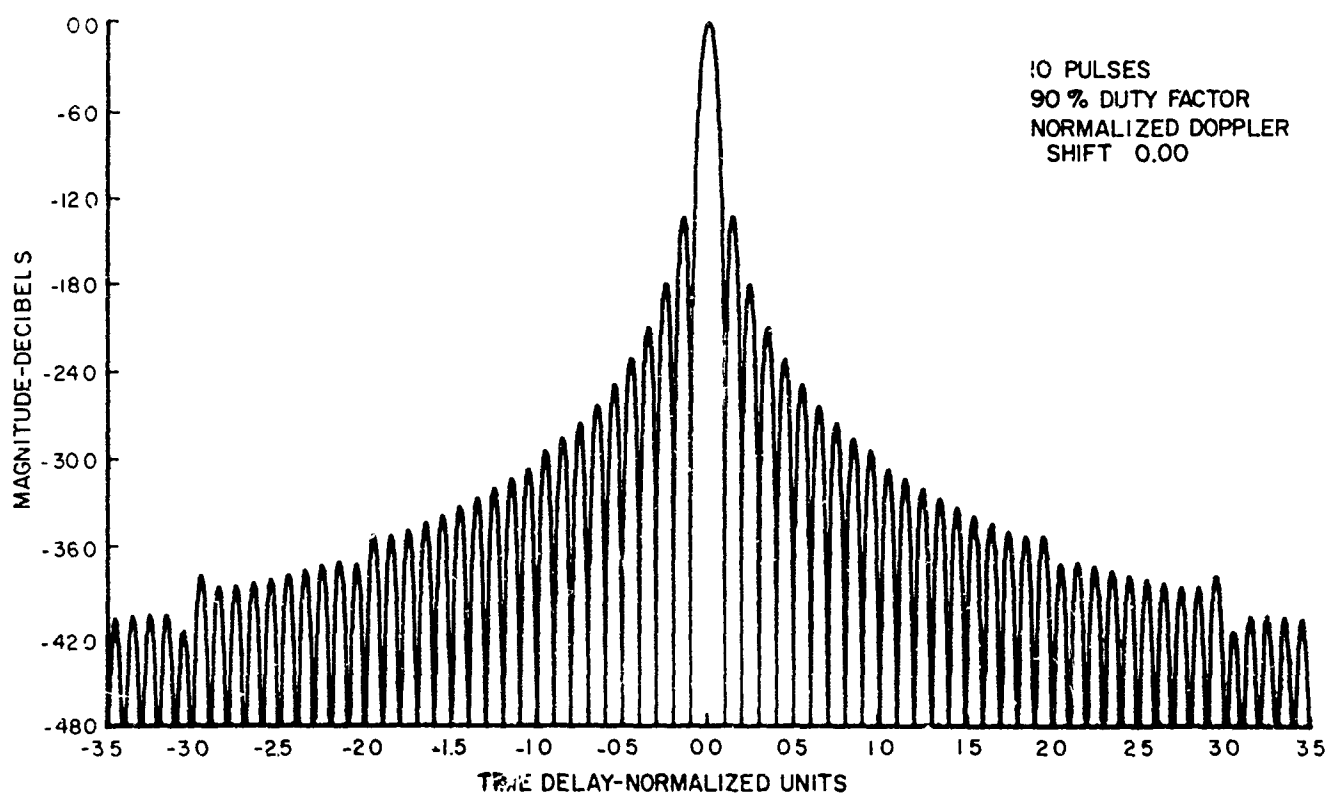


Figure 8 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

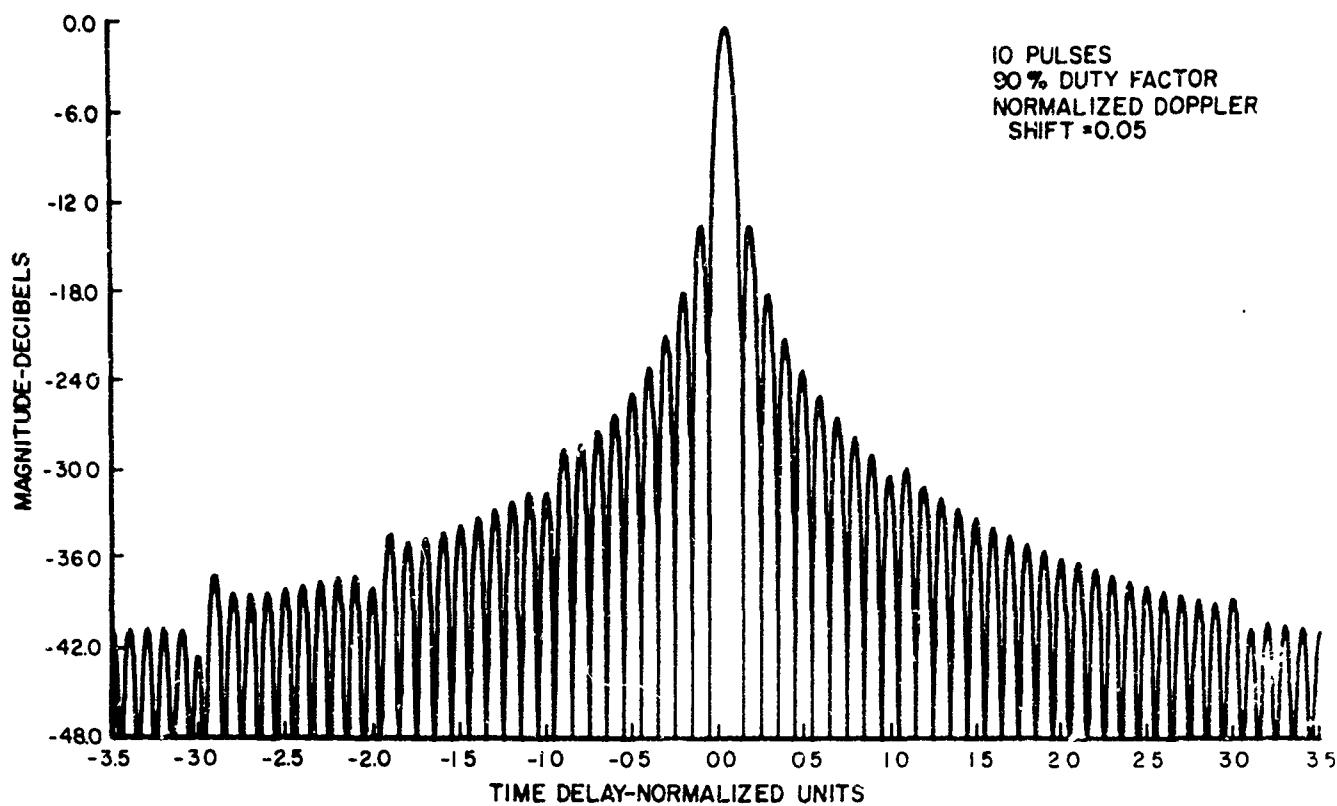


Figure 8 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

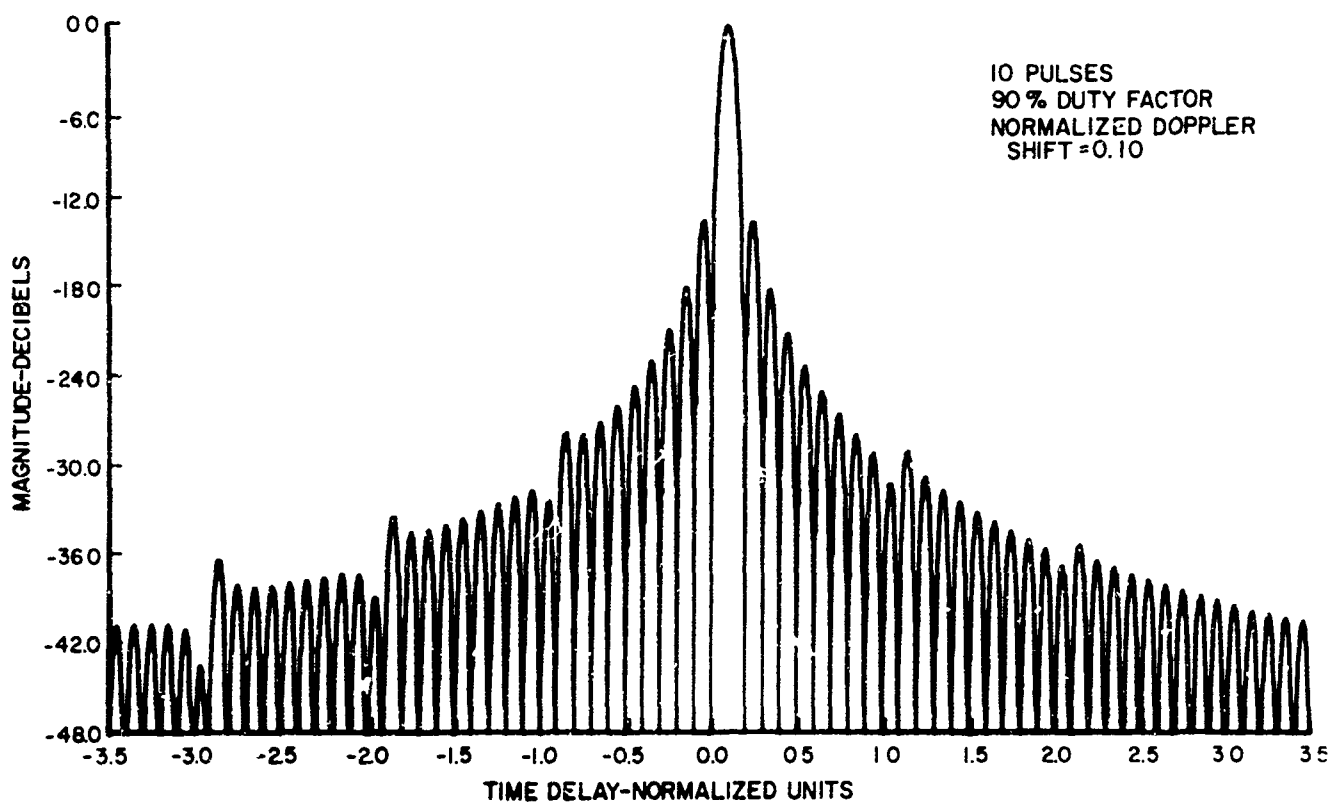


Figure 8 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES



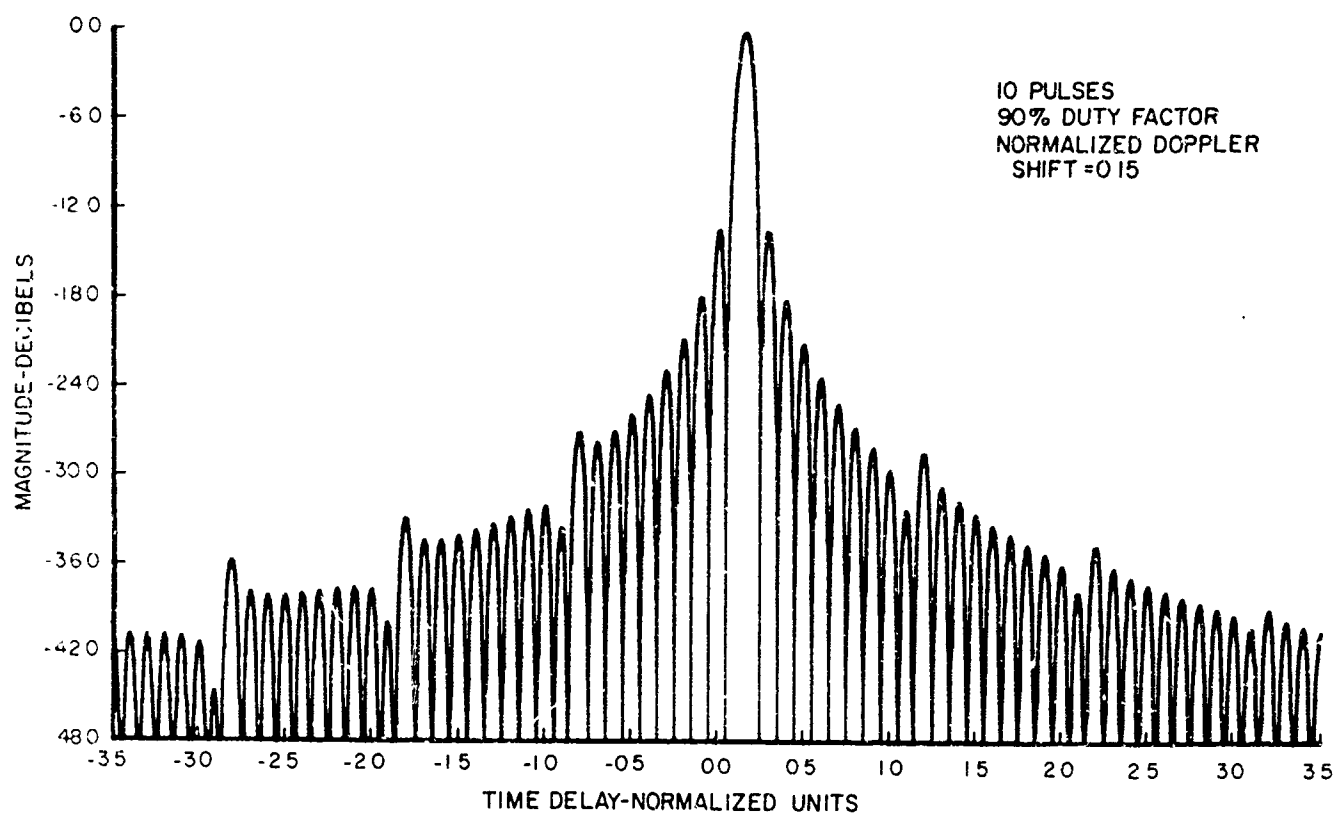


Figure 8 (d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

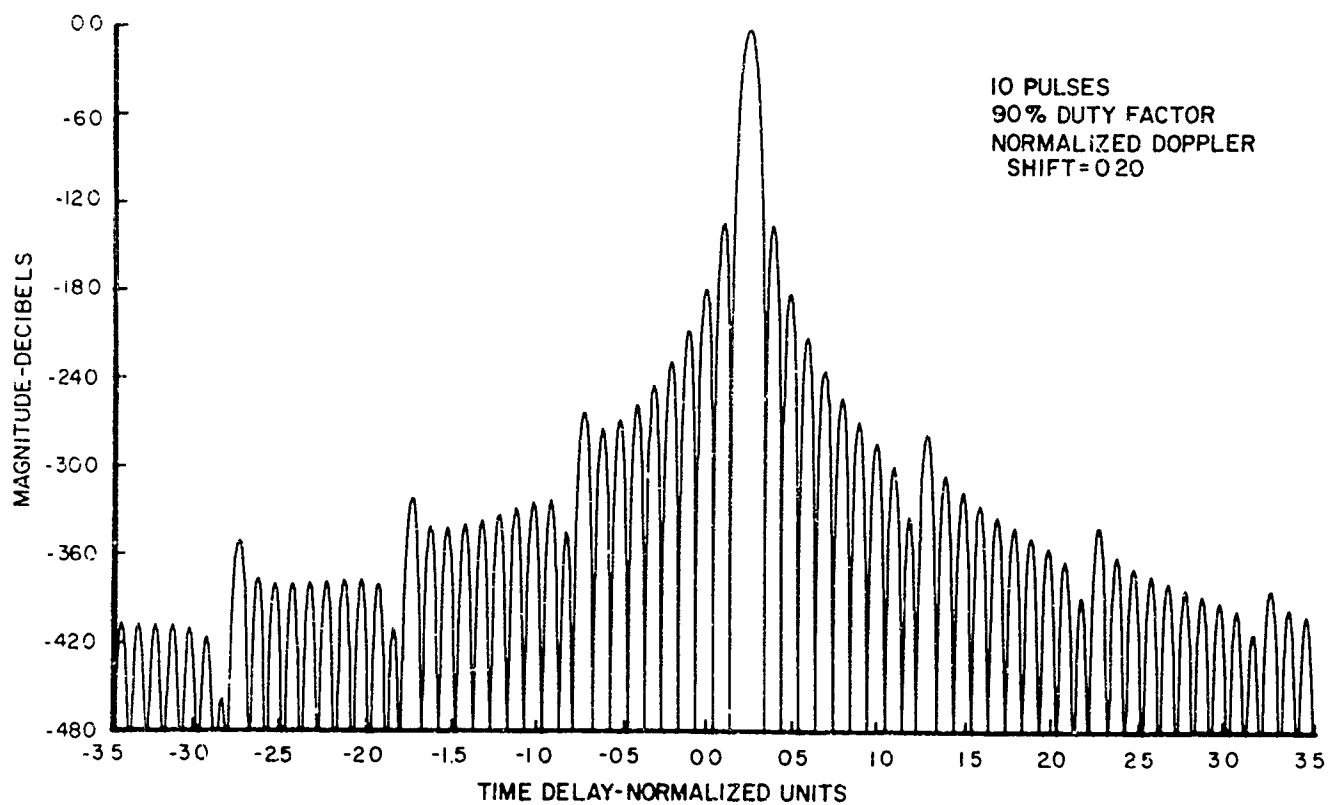


Figure 8 (e.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

regard. since the third grating lobe (-29.5db) is larger than the second (-30.4 db). However, the next six grating lobes, starting with the fourth, are -28.0 db, -26.9 db, -26.4 db, -26.8 db, -28.9 db, and -34.0 db respectively, so that the farther out grating lobes never get very high, being suppressed by the fall off of the chirp sub-pulse delay sidelobes.

The ambiguity function for a train of three chirp sub-pulses with a 50% duty factor is shown in figure 9. In this case, the doppler cuts are spaced at normalized doppler intervals,  $\Delta V' = 1/6$ , which is  $10/3$  as far apart as in the 10 pulse case. However, if equal time duration signals were used in the two cases,  $T$  would also be  $10/3$  great in the three pulse case, so that in both cases the cuts represent the same amount of actual doppler shift, for equal time duration signals. Comparing the zero-doppler response, figure 9(a), with the corresponding 10 pulse case, figure 7(a), we see that the 3 pulse case is slightly better. This is probably due to the fact that the sub-pulse time-bandwidth product is  $10/3$  larger, making its zeroes more nearly equally spaced. However, the 3 pulse response deteriorates more rapidly with increasing doppler shift than does the 10 pulse response. In particular, the first grating lobe to the left of the main lobe is incompletely suppressed. This seems to be due to the closer spacing of the grating lobes relative to their width.

Finally, a train of 30 chirp sub-pulses with a 50% duty factor was considered. Because of plotter limitations, the ambiguity functions are in two parts. Figure 10 contains the positive values of delay and figure 11 the negative values of delay. The doppler cuts have a spacing  $\Delta\nu' = 1/60$ , which, by the argument used previously, is the same spacing in absolute doppler as used for the 3 and 10 pulse cases, for equal signal durations. We see that the grating lobe ambiguities along the delay axis are suppressed, although not as well as for the 3 and 10 pulse cases. However, the doppler tolerance is better than for 3 or 10 pulses. Again, as in figure 6(a) the grating lobes shown increase in amplitude. The first, second, and third grating lobes along the delay axis (figure 10a), are -32.4 db, -28.7 db, and -25.7 db respectively. The next six grating lobes, not plotted, are -24.1 db, -24.0 db, -26.5 db, -35.7 db, -34.8 db, and -29.1 db, so that they do not rise too high before being suppressed by the fall off of the chirp sub-pulse delay sidelobes.

To summarize, it appears from the limited number of cases considered that grating lobes along the delay axis can be suppressed by this technique for either a small or large number of pulses in the train. The suppression along the delay axis works somewhat better for small numbers of sub-pulses, but the

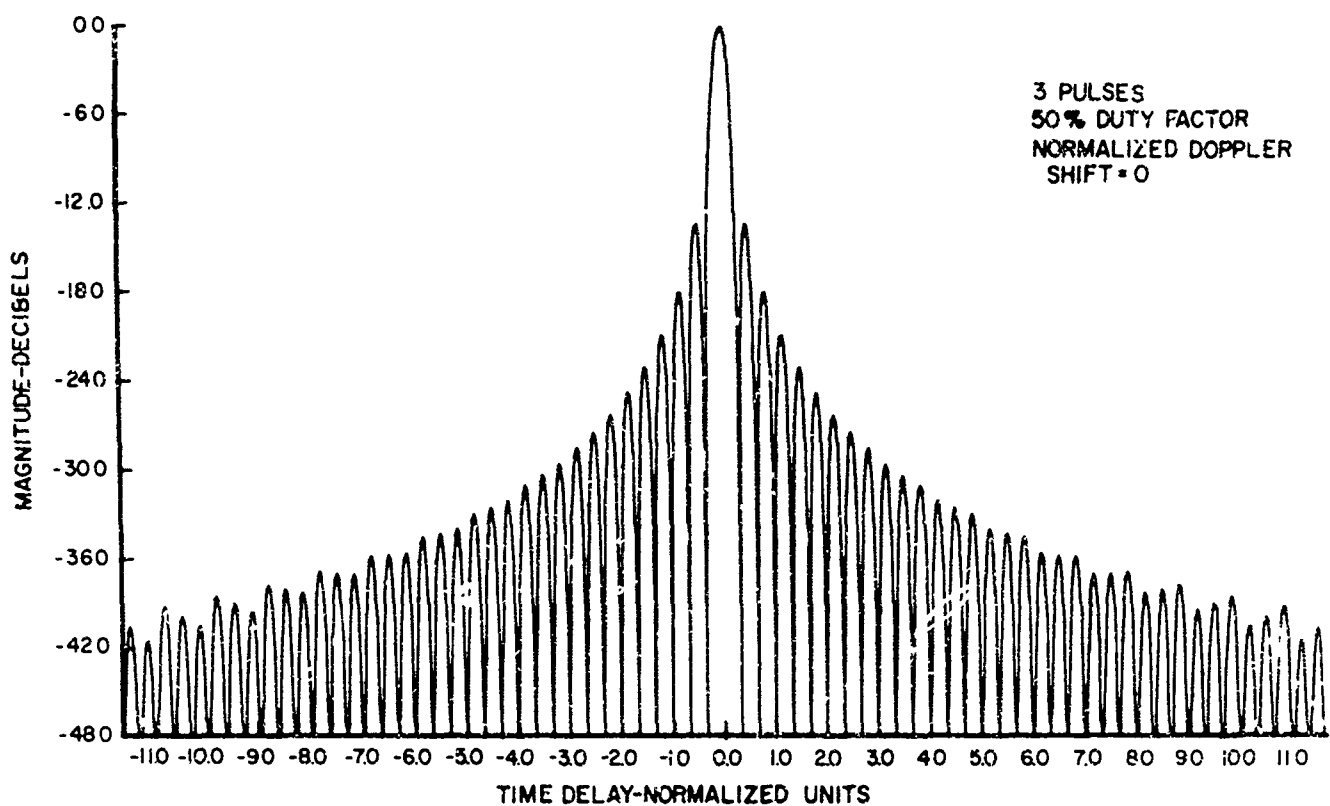


Figure 9 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

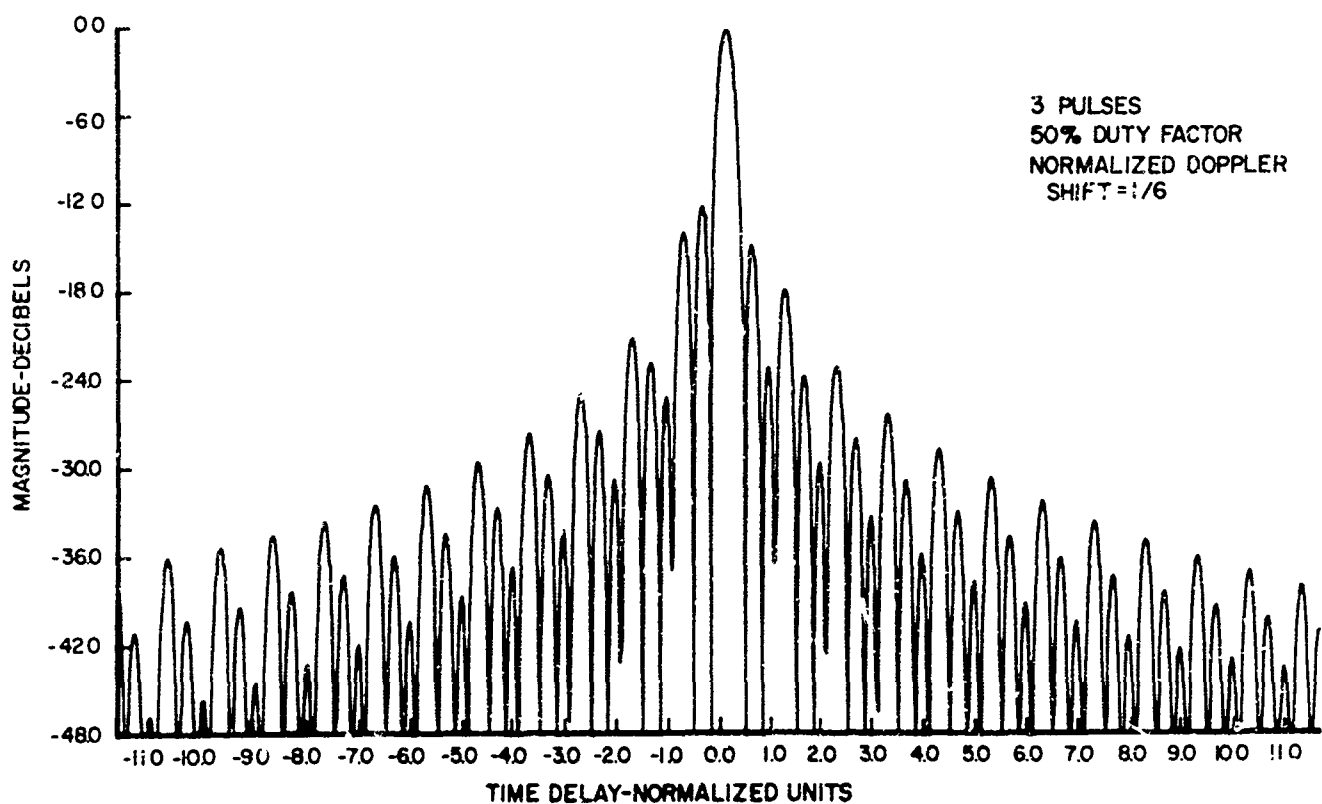


Figure 9 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

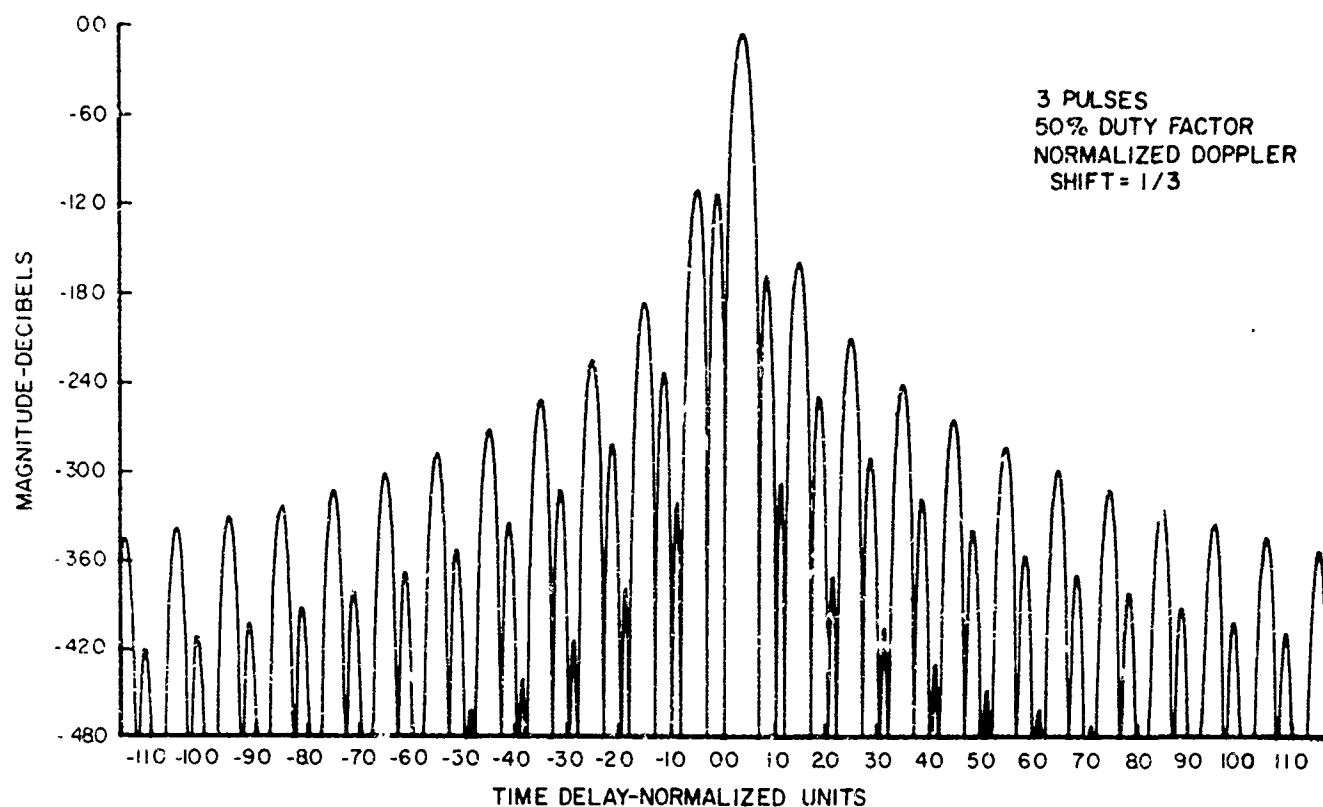


Figure 9 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

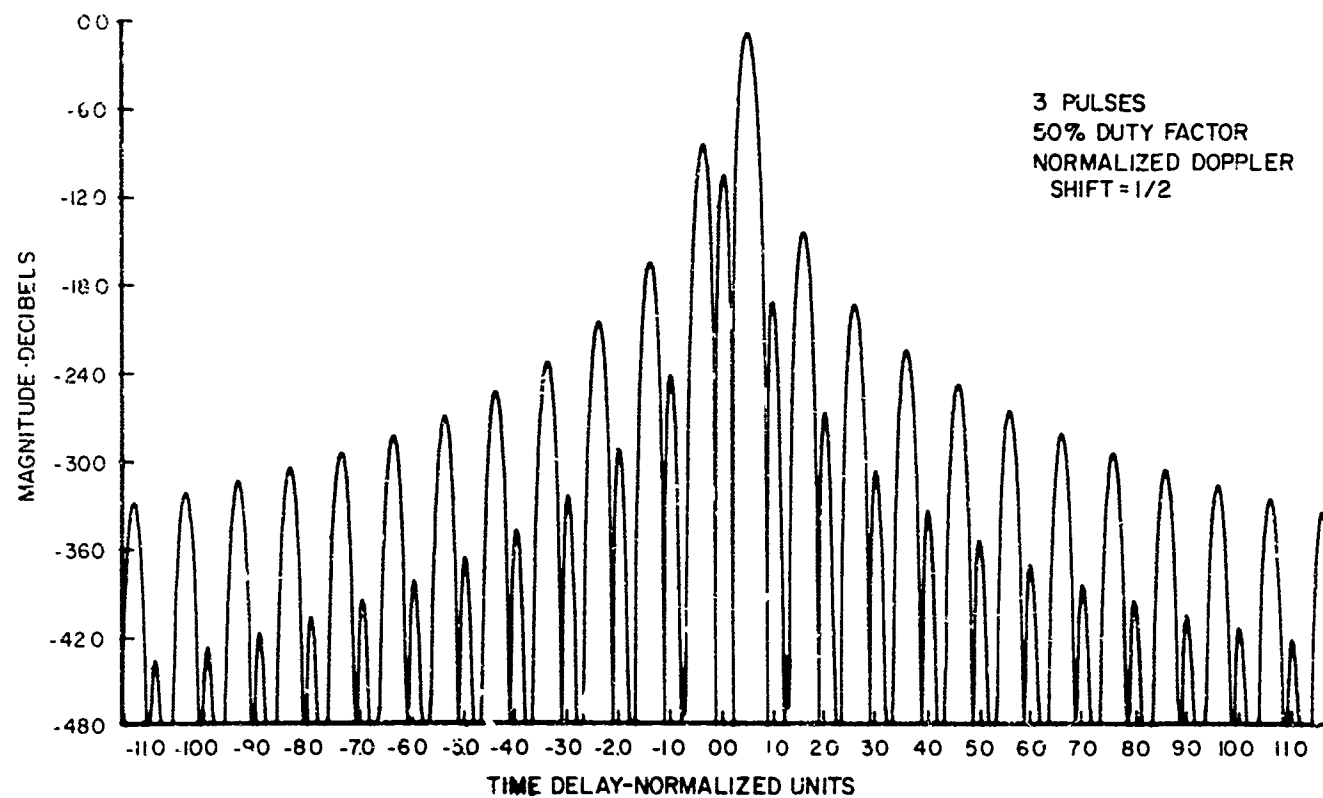


Figure 9 (d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

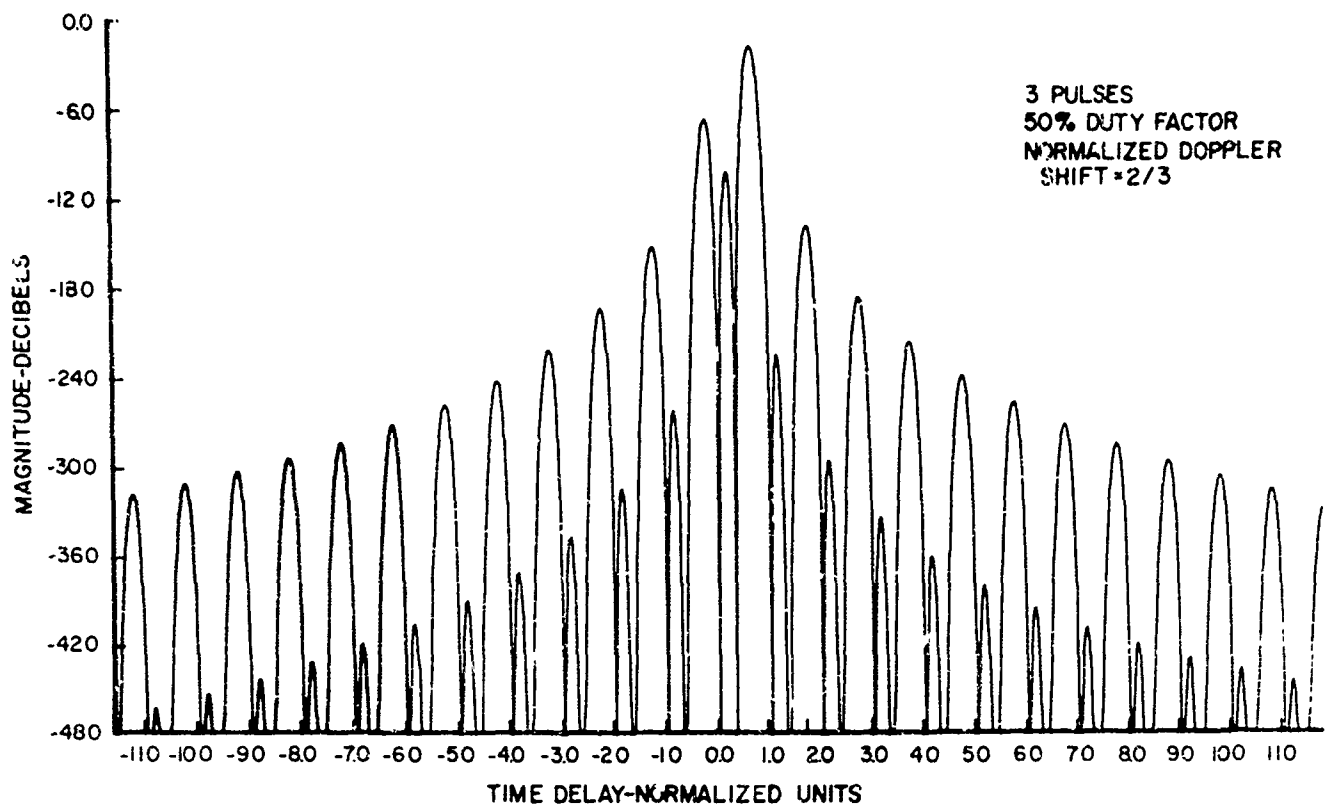


Figure 9 (e.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

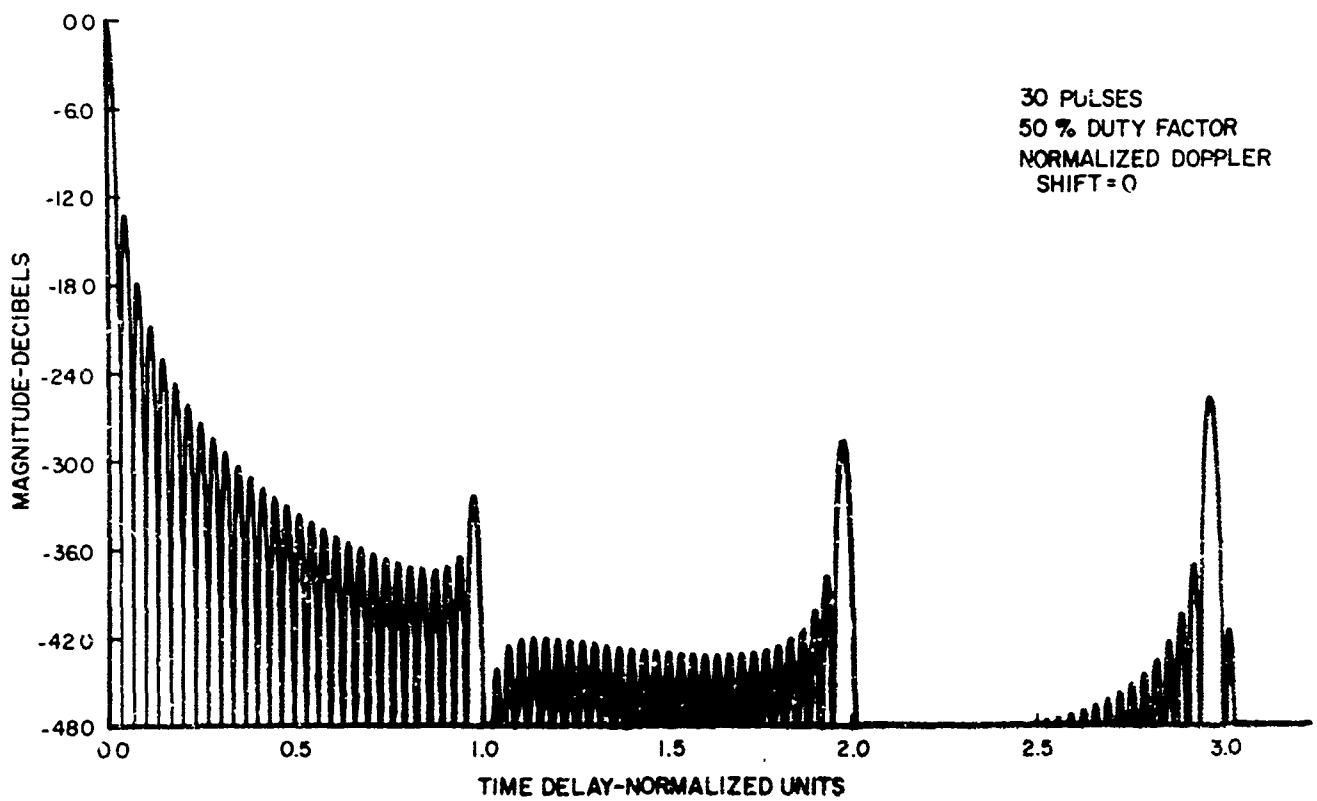


Figure 10 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

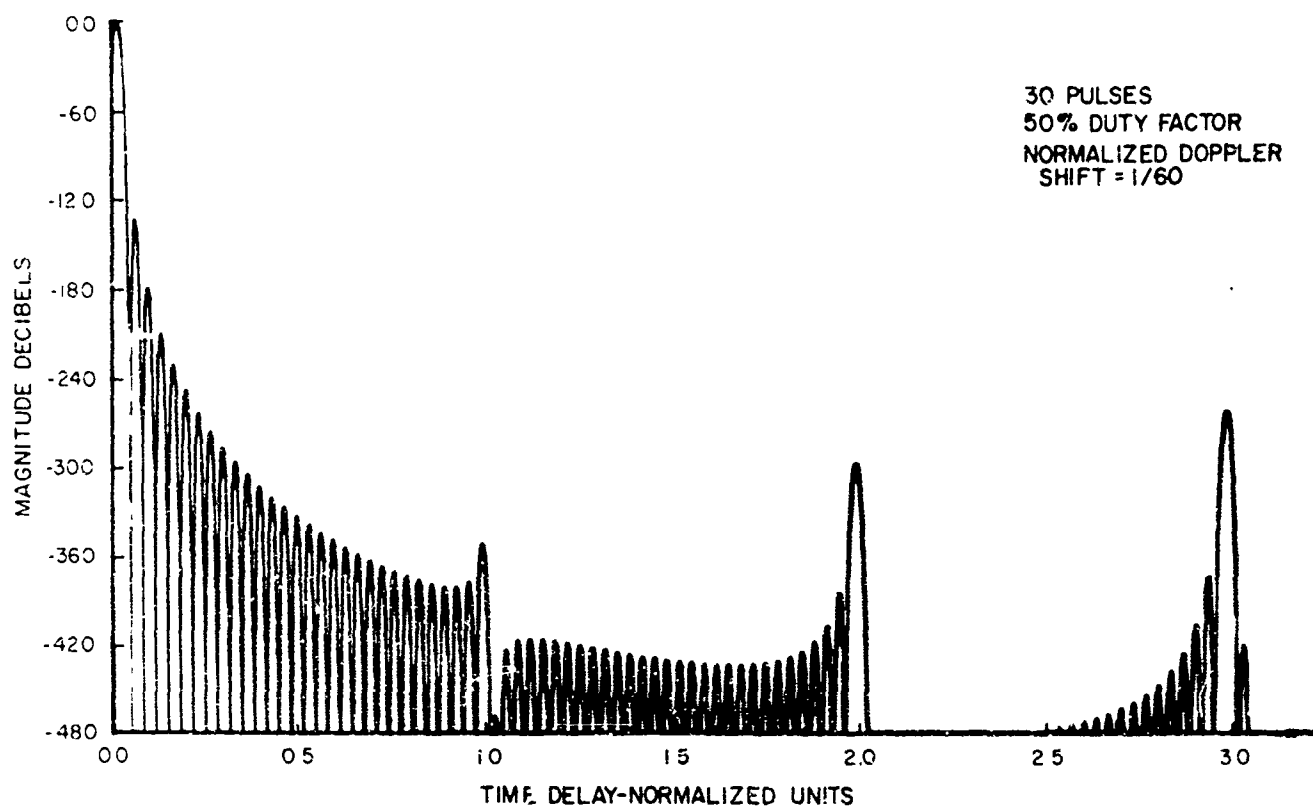


Figure 10 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

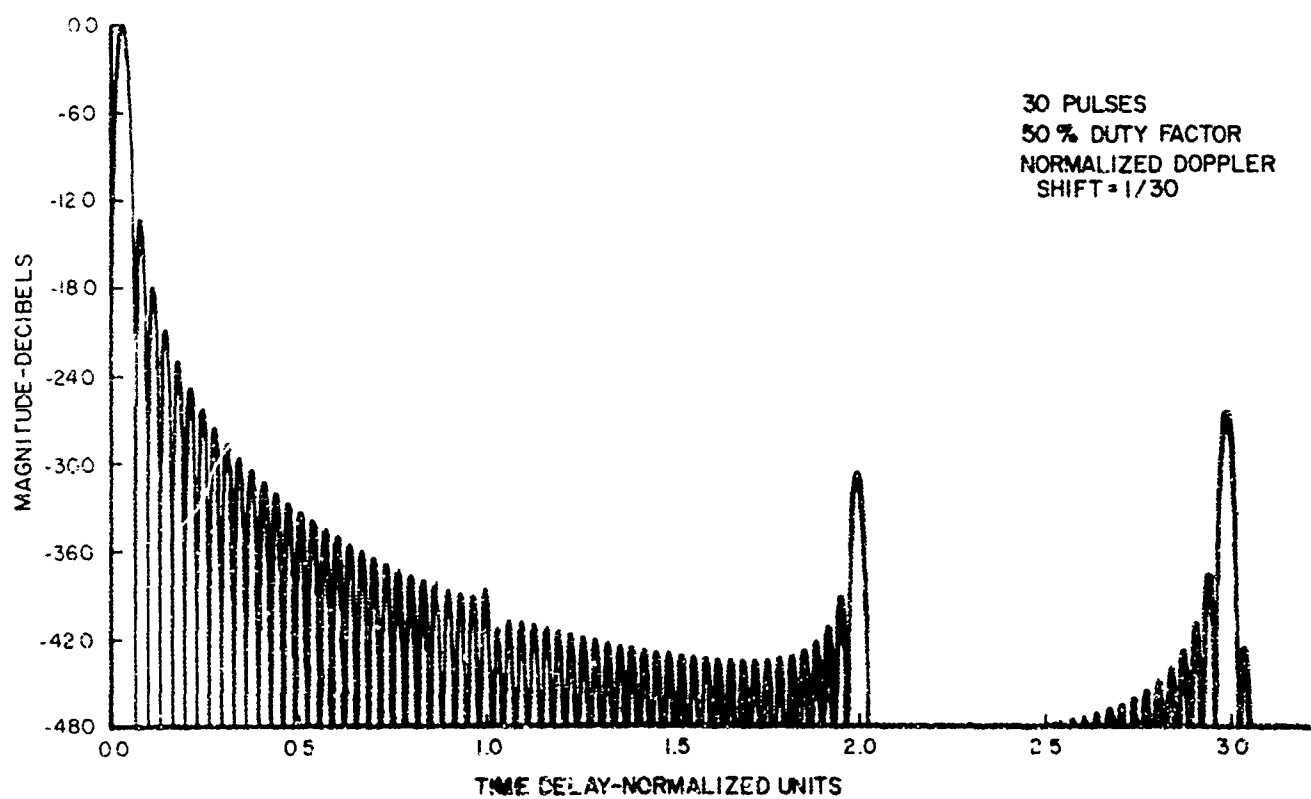


Figure 10 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

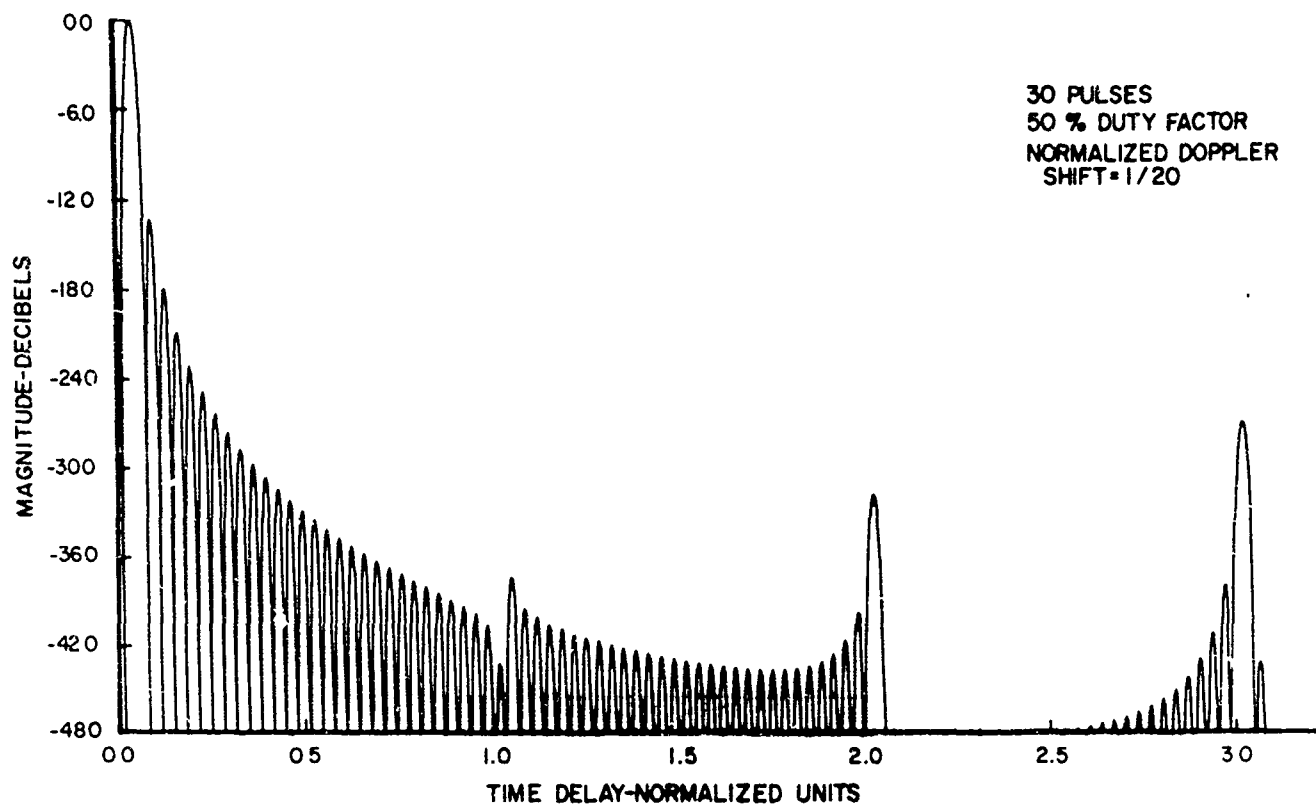


Figure 10 (d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

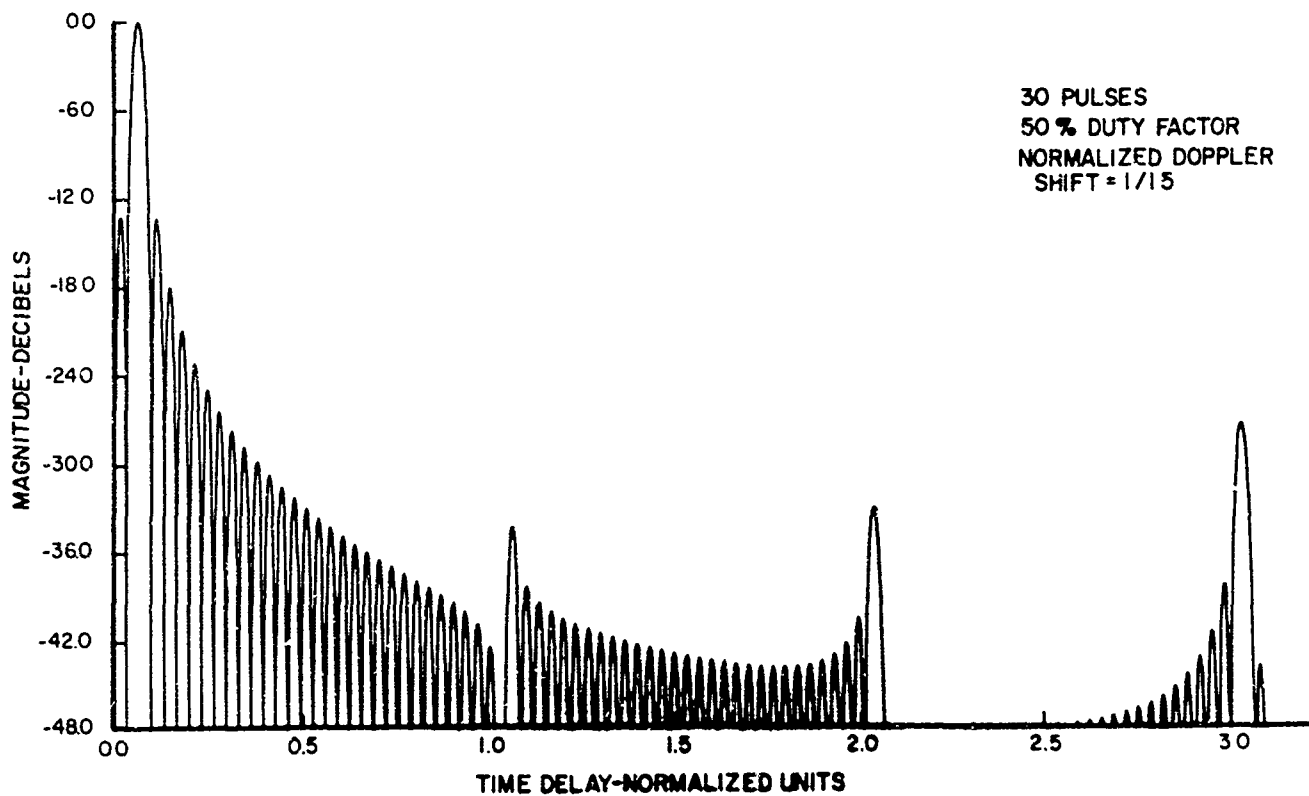


Figure 10 (e.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES



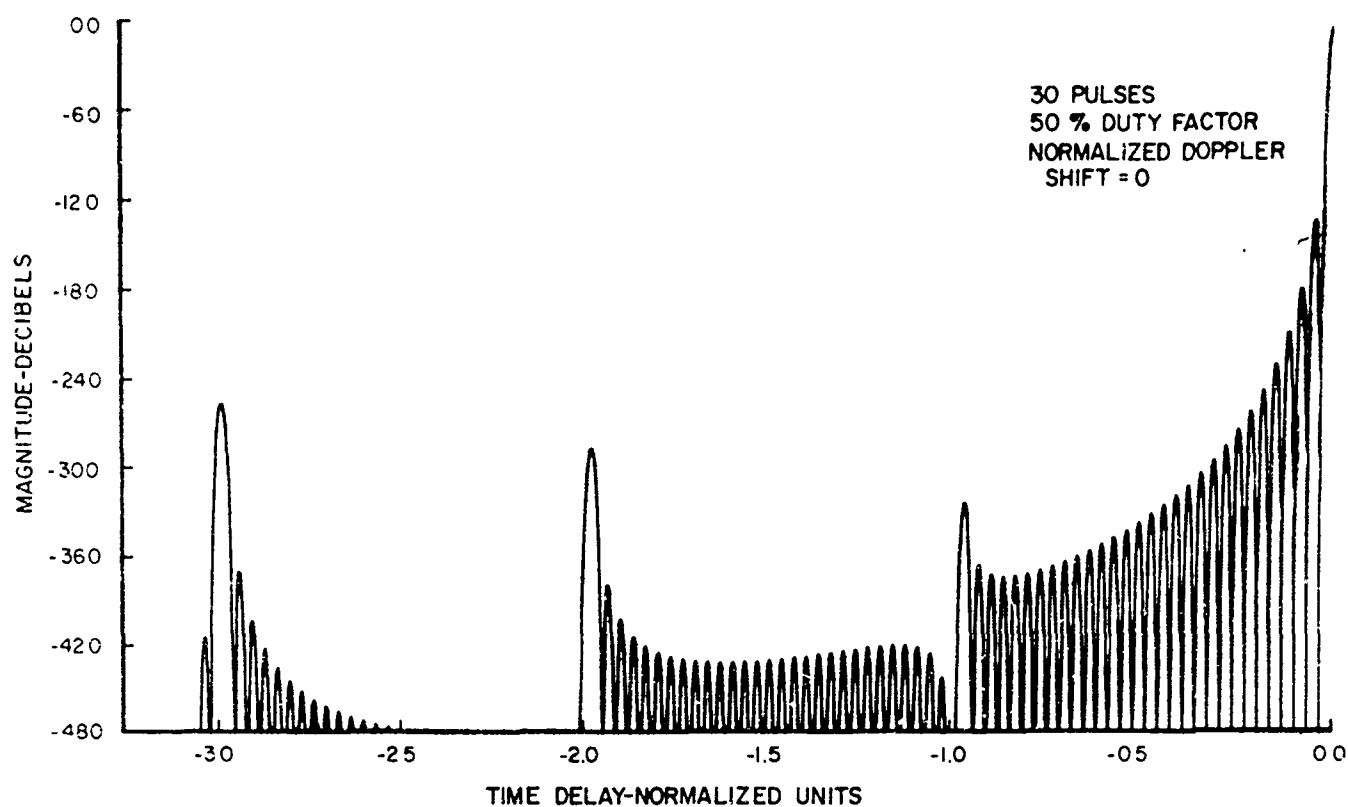


Figure 11 (a.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

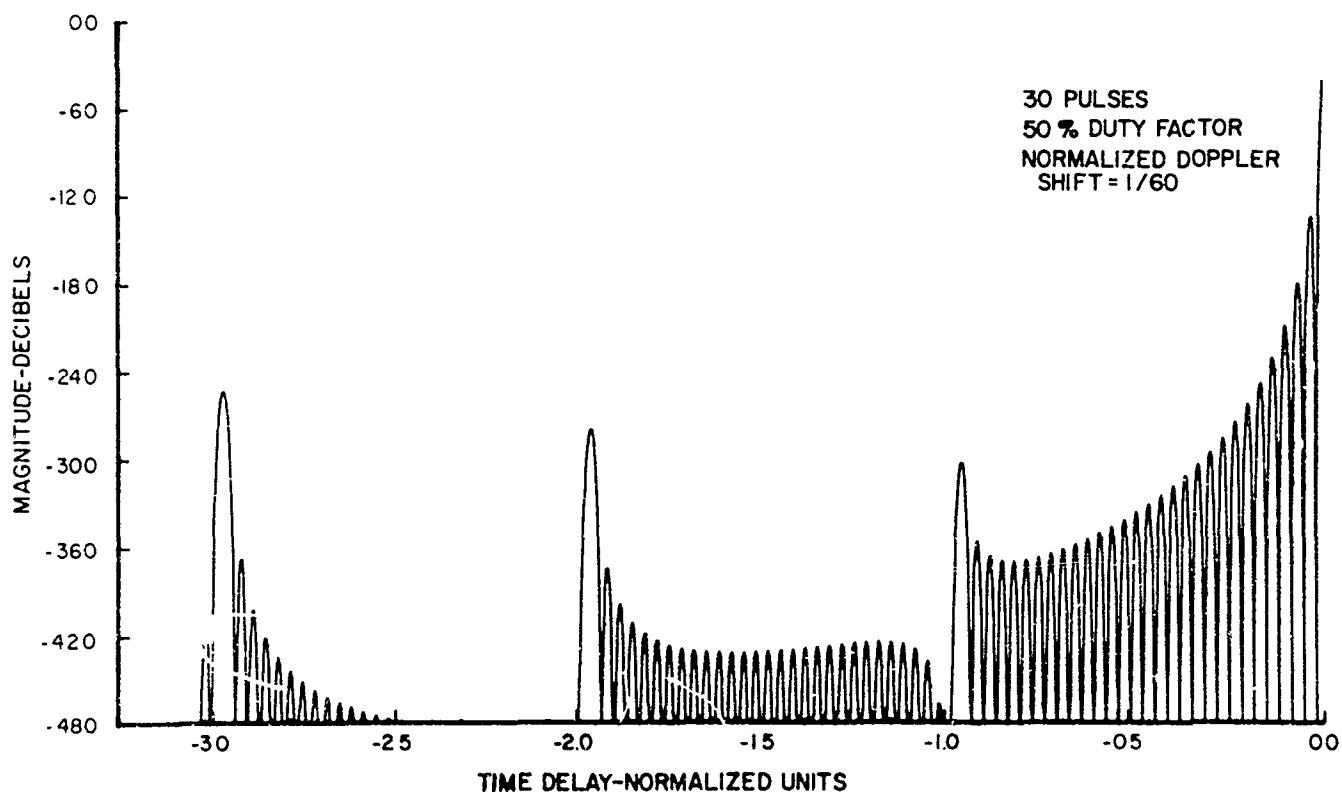


Figure 11 (b.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

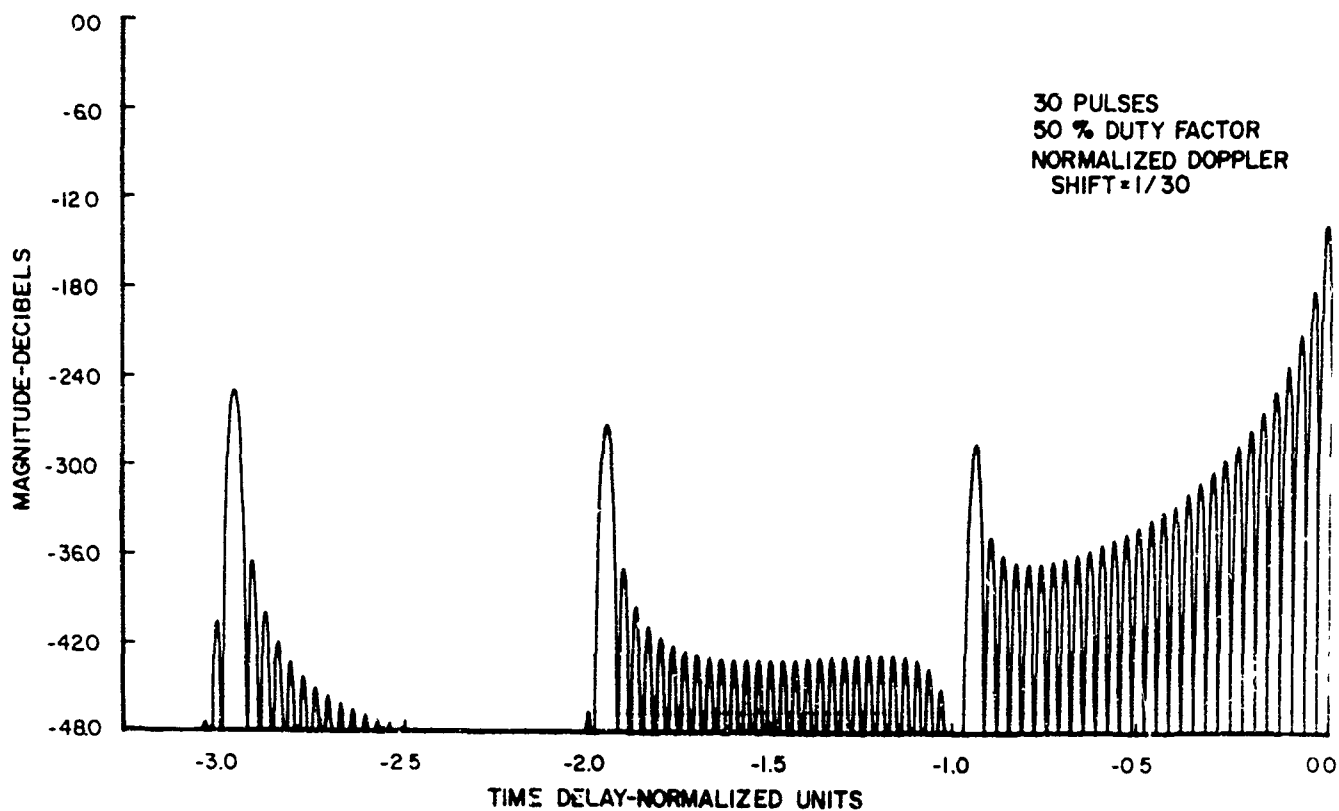


Figure 11 (c.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

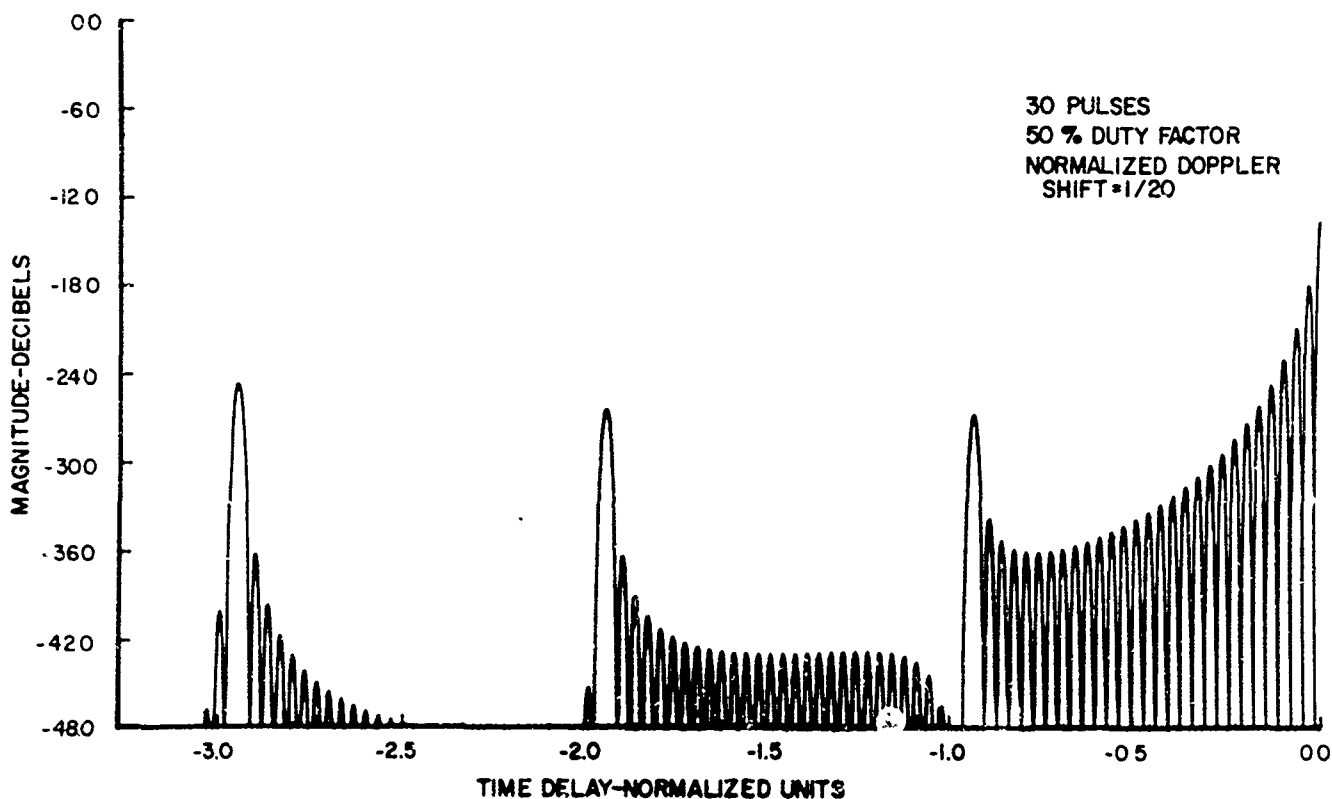


Figure 11 (d.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

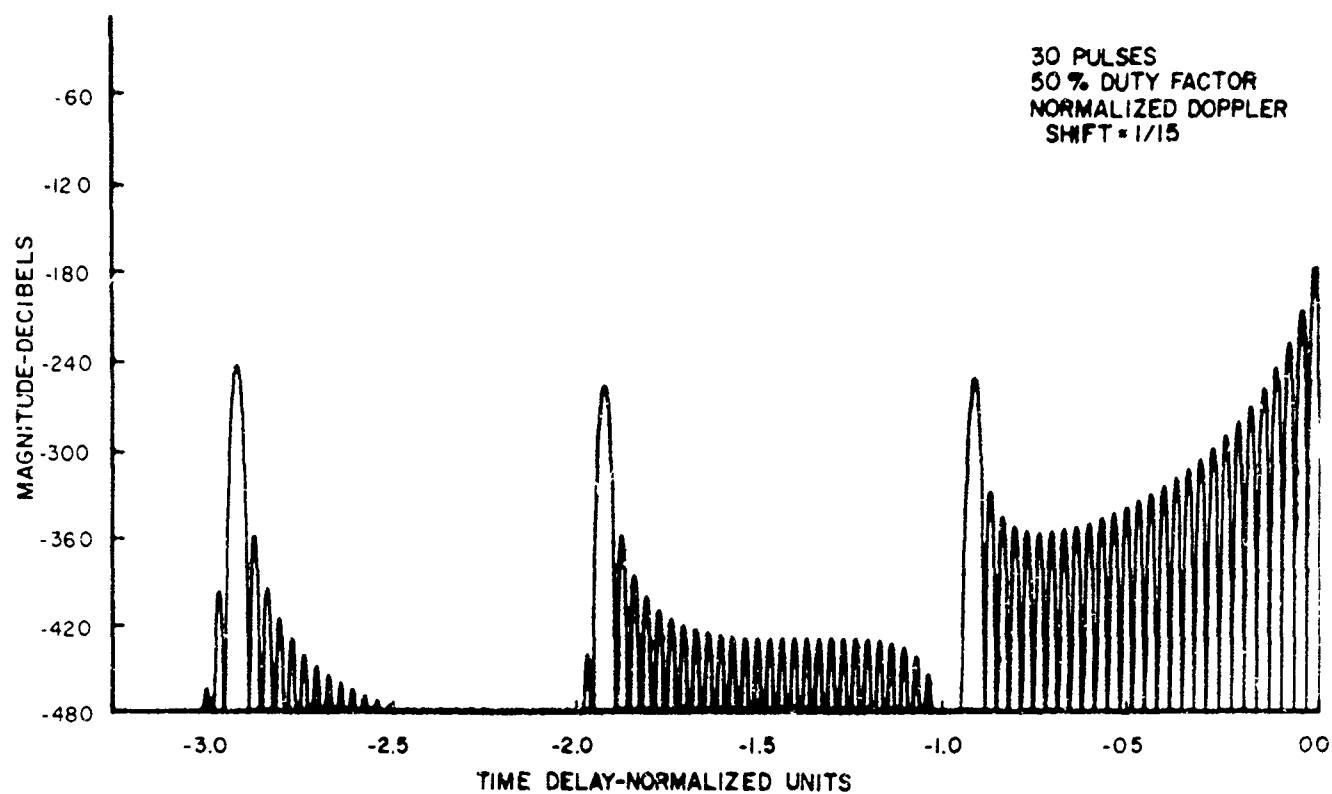


Figure 11 (e.) AMBIGUITY FUNCTION WITH CHIRP SUB-PULSES

number is not critical. The suppression along the delay axis also improves with duty factor, but this is not critical either. The doppler tolerance - -the range of doppler shifts over which the time response remains substantially undistorted-- is strongly dependent upon the duty factor, on the other hand, becoming much larger as the duty factor of the pulse train increases. The doppler tolerance also increases as the number of pulses increases, but the dependence is not as strong as upon the duty factor.

#### SECTION IV

##### VALIDITY OF THE RESOLUTION CRITERION

In using Woodward's ambiguity function as a resolution criterion, one is limited by two assumptions implicit in it. These are:

- 1) Range acceleration and higher derivatives are assumed to have negligible effect upon the signal.
- 2) The doppler effect is approximated as a carrier frequency translation, with no distortion of the signal modulation function.

The first assumption introduces no problems. If we assume as a worst case a one millisecond signal time duration and 100 g's acceleration, then from

$$\Delta R = \frac{R}{2} \Delta t^2 \quad (13)$$

one obtains a range change during the signal duration due to radial acceleration of only .0016 feet which is negligible at microwave wavelengths. The second assumption, however, limits the range of doppler shifts for which the calculation is valid as a resolution criterion. Rihaczek [3] has derived an equation which gives the maximum range rate for which

assumption (2) holds as

$$\dot{R} \leq \frac{0.1 C}{(\beta\tau)_{\text{overall}}} \quad (14)$$

which may be expressed in terms of doppler shift as

$$\nu = \frac{2\dot{R}}{C} f_o \leq \frac{.2f_o}{(\beta\tau)_{\text{overall}}} \quad (15)$$

Using the fact that the overall time duration of the signal is  $NT$ , defining  $\gamma$  to be the fractional bandwidth of the signal, and substituting  $\nu' = T\nu$ , equation (15) may be transformed to

$$\nu' \leq \frac{0.2}{\gamma N} \quad (16)$$

This restriction was observed, with an assumed 10 per cent bandwidth, in computing the ambiguity functions shown in Section III. One should interpret the doppler shift in these ambiguity functions not as the total target doppler shift, but as the difference between the target doppler shift and the

doppler shift to which the receiver is matched. Furthermore, for these ambiguity functions to represent a valid resolution criterion, it is necessary that the receiver be matched not to the transmitted signal, but to the expected received signal. That is, the receiver (matched filter) must be compensated not only for the expected doppler shift of the carrier frequency, but for the time dilation or spectrum spreading distortion of the signal modulation function corresponding to the expected doppler shift. Thus, the modulation function distortion which is considered negligible in these calculations is that due to the incremental doppler shift of the target, not that due to the total doppler shift.

## SECTION V

### CONCLUSIONS

By chirp coding each sub-pulse in the train, the utility of a linear frequency-stepped pulse train is greatly enhanced. The overall time-bandwidth product which can be used, subject to the condition of an unambiguous time response, is increased by a factor equal to the sub-pulse time-bandwidth product. Furthermore, acceptable time responses can be obtained with either a small or large number of sub-pulses, and with small or large pulse train duty factors. The doppler tolerance of the signal waveform, however, depends strongly upon the intra-train duty factor, increasing as duty factor increases. These design characteristics are quite different from those encountered with time and frequency staggered pulse trains, which require large numbers of sub-pulses and small duty factors to obtain low time sidelobes, and have little doppler tolerance.

The ambiguity functions illustrated have moderately high sidelobe levels, which are undesirable in some applications. Lower sidelobes might be obtained by applying well-known amplitude weighting techniques to these waveforms. The close-in time sidelobes, which now fall off as  $\sin Nx / \sin x$ , could be greatly reduced by weighting each pulse in the train according to a Taylor weighting function, for example. The magnitude of



the partially suppressed grating lobes might be reduced by lowering the time sidelobe level of the chirp sub-pulse response. This can be done by amplitude weighting the chirp signal in time or frequency, as is well known. However, one should be careful to select a weighting function which gives equally or nearly equally spaced zeroes along the delay axis, in order to properly suppress the grating lobes.

## APPENDIX - DERIVATION OF AMBIGUITY FUNCTION

All the results obtained in this paper depend upon the ambiguity function of the linear frequency-stepped pulse train given in equation (1). Although this has already been derived in Reference [1], we will derive it here also, in order to make clear what assumptions and/or approximations are used in extending this equation to a situation where chirp coding is used on each pulse.

The general expression for the ambiguity function is

$$x(\tau, \nu) = \frac{1}{2E} \int_{-\infty}^{\infty} \mu(t) \mu^*(t-\tau) e^{j2\pi\nu t} dt \quad (A-1)$$

where

- E = total signal energy
- $\mu(t)$  = complex modulation function of the signal
- $\tau$  = incremental time delay
- $\nu$  = incremental doppler shift

Assume a train of N equally spaced pulses, with identical amplitude modulation, but pulse-to-pulse variation of the phase modulation

$$\mu(t) = \sum_{n=0}^{N-1} a(t-nT) e^{j\phi_n(t-nT)} \quad (A-2)$$

where  $a(t)$  = amplitude modulation of all pulses

$\phi_n(t)$  = phase modulation of the  $n$ th pulse

$T$  = repetition interval

Substituting (A-2) in (A-1) gives

$$x(\tau, \nu) = \sum_{l=0}^{N-1} \sum_{n=0}^{N-1} F_{l,n}(\tau, \nu) \quad (A-3)$$

$$F_{l,n}(\tau, \nu) = \frac{1}{2E} \int_{-\infty}^{\infty} a(t-lT) e^{j\phi_l(t-lT)} a(t-\tau-nT) e^{-j\phi_n(t-\tau-nT)} e^{j2\pi\nu t} dt \quad (A-4)$$

The summation in equation (A-3) can be broken up into three summations, the purpose of which will become clear as we proceed.

$$x(\tau, \nu) = S_1 + S_2 + S_3 \quad (A-5)$$

$$S_1 = \sum_{n=0}^{N-1} F_{n,n}(\tau, \nu) \quad (A-6)$$

$$S_2 = \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} F_{n, n+m}(\tau, \nu) \quad (A-7)$$

$$S_3 = \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} F_{n+m, n}(\tau, \nu) \quad (A-8)$$

If one regards the double sum in equation (A-3) as a matrix, then  $S_1$  contains the terms on the main diagonal,  $S_2$  contains the terms above the main diagonal, and  $S_3$  the terms below the main diagonal. The indices  $n$  and  $m$  run along and across diagonals, respectively, rather than along rows and columns.

By making the substitutions,  $t - nT = t'$  in  $S_1$  and  $S_2$ , and  $t - (n+m)T = t'$  in  $S_3$ , one can obtain

$$S_1 = \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi nT\nu} x_{n,n}(\tau, \nu) \quad (A-9)$$

$$x_{n+m,n}(\tau, \nu) = \frac{1}{2E_1} \int_{-\infty}^{\infty} a(t') e^{j\phi_n + m(t')} a(t' \cdot \tau + mT) e^{-j\phi_n(t' - \tau + mT)} e^{j2\pi\nu t'} dt' \quad (A-14)$$

The total signal energy,  $E$ , has been replaced by  $NE_1$ , where  $E_1$  is the energy of each pulse. If we let  $\phi_n(t) = \phi(t) - a_n 2\pi F t$ , where  $a_n$  is an integer between 0 and  $N-1$  and  $\{a_n\}$  is the frequency shifting code, we can rewrite equations (A-9) through (A-14) as

$$S_1 = \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi n T \nu} e^{-j2\pi a_n F \tau} x_p(\tau, \nu) \quad (A-15)$$

$$S_2 = \frac{1}{N} \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} e^{j2\pi n T \nu}$$

$$e^{-j2\pi a_{n+m} F(\tau + mT)} x_p(\tau + mT, \nu + [a_{n+m} - a_n]F) \quad (A-16)$$

$$x_{n,n}(\tau, \nu) = \frac{1}{2E_1} \int_{-\infty}^{\infty} a(t') e^{j\phi_n(t')} a(t'-\tau) e^{-j\phi_n(t'-\tau)} e^{j2\pi\nu t'} dt' \quad (A-10)$$

$$S_2 = \frac{1}{N} \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} e^{j2\pi n T \nu} x_{n,n+m}(\tau, \nu) \quad (A-11)$$

$$x_{n,n+m}(\tau, \nu) = \frac{1}{2E_1} \int_{-\infty}^{\infty} a(t') e^{j\phi_n(t')} a(t'-\tau-mT) e^{-j\phi_{n+m}(t'-\tau-mT)} e^{j2\pi\nu t'} dt' \quad (A-12)$$

$$S_3 = \frac{1}{N} \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} e^{j2\pi(n+m)T\nu} x_{n+m,n}(\tau, \nu) \quad (A-13)$$

$$S_3 = \frac{1}{N} \sum_{m=1}^{N-1} \sum_{n=0}^{N-1-m} e^{j2\pi(n+m)TV} e^{-j2\pi a_n F(\tau-mT)} x_p(\tau-mT, \nu - [a_{n+m} - a_n]F) \quad (A-17)$$

$$x_p(\tau, \nu) = \frac{1}{2E_1} \int_{-\infty}^{\infty} a(t) e^{j\phi(t)} a(t-\tau) e^{-j\phi(t-\tau)} e^{j2\pi\nu t} dt \quad (A-18)$$

where the function  $x_p(\tau, \nu)$  is the ambiguity function of a single component pulse.

Now specialize the frequency code to a linear frequency step, letting  $a_n = n$ , and perform the summations over  $n$  by using an identity given by Guillemin [4]

$$\frac{1-e^{jkX}}{1-e^{jX}} \equiv \sum_{n=0}^{k-1} e^{jnX} \quad (A-19)$$

to obtain the following expressions.

$$S_1 = \frac{x_p(\tau, \nu)}{N} \cdot \frac{1 - e^{j2\pi N[TV-F\tau]}}{1 - e^{j2\pi[TV-F\tau]}} \quad (A-20)$$

$$S_2 = \frac{1}{N} \sum_{m=1}^{N-1} x_p(\tau + mT, \nu + mF) \frac{1 - e^{j2\pi[N-m][TV-F(\tau+mT)]}}{1 - e^{j2\pi[TV-F(\tau+mT)]}} \cdot e^{-j2\pi mF(\tau+mT)} \quad (A-21)$$

$$S_3 = \frac{1}{N} \sum_{m=1}^{N-1} x_p(\tau - mT, \nu - mF) \frac{1 - e^{j2\pi[N-m][TV-F(\tau-mT)]}}{1 - e^{j2\pi[TV-F(\tau-mT)]}} \cdot e^{j2\pi mF\tau} \quad (A-22)$$

By making suitable manipulations of the exponential terms, replacing  $m$  by  $-m$  in  $S_2$ , and substituting in equation (A-5), one obtains



$$\begin{aligned}
x(\tau, \nu) = \frac{1}{N} \sum_{m=-(N-1)}^{N-1} x_p(\tau-mT, \nu-mF) \cdot \frac{\sin \pi[N-|m|][TV-F(\tau-mT)]}{\sin \pi[TV-F(\tau-mT)]} \\
\cdot e^{j\pi[N+m-1]TV} e^{-j\pi[N-m-1]F[\tau-mT]} \quad (A-23)
\end{aligned}$$

$$|x(\tau, \nu)|^2 = \frac{1}{N^2} \sum_{m=-(N-1)}^{N-1} \sum_{n=-(N-1)}^{N-1} x_p(\tau-mT, \nu-mF) x_p^*(\tau-nT, \nu-nF)$$

$$\frac{\sin \pi[N-|m|][TV-F(\tau-mT)]}{\sin \pi[TV-F(\tau-mT)]} \cdot \frac{\sin \pi[N-|n|][TV-F(\tau-nT)]}{\sin \pi[TV-F(\tau-nT)]}$$

$$e^{j\pi(m-n)TV} e^{j\pi[N-1][m-n]FT} e^{j\pi(m-n)F[\tau-(m-n)T]} \quad (A-24)$$

Up to this point, the derivation has been completely general, with no restrictions on the ambiguity function of a component pulse,  $x_p(\tau, \nu)$ , or on the time and frequency steps,  $T$  and  $F$ . However, in order to reduce equation (A-24) to the desired form, equation (1), it is necessary to impose the condition

$$x_p(\tau-mT, \nu-mF) x_p^*(\tau-nT, \nu-nF) = \delta_{mn} |x_p(\tau-mT, \nu-mF)|^2 \quad (A-25)$$

where  $\delta_{mn}$  is the Kronecker delta.

$$\begin{aligned} \delta_{mn} &= 1, m=n \\ &= 0, m \neq n \end{aligned} \quad (A-26)$$

This means that there must be no overlap between the functions,  $x_p(\tau-mT, \nu-mF)$ , for different  $m$ 's. If one refers again to figure 1(b), which shows these periodically shifted functions (major ambiguities) it can be seen that complete absence of overlap is achieved if  $x_p(\tau, \nu)$  is either delay limited to an interval  $(-T/2, T/2)$  or doppler limited to an interval  $(-F/2, F/2)$ .

For the case of monotone sub-pulses, the former condition will be obtained if the sub-pulse length,  $T'$ , is less than  $T/2$ , which can be seen from equation (2). This requires the pulse train duty factor,  $\delta$ , to be less than or equal to 0.5. When  $\delta$  is greater than 0.5, time overlap between shifted  $x_p$  functions can occur. However, if  $F \gg 1/T$ , which is true for the monotone sub-pulse cases we have computed, a doppler overlap will take place

only in the far out sidelobes, and equation (1) will still be a good approximation.

With chirp coding on each sub-pulse, the sub-pulse ambiguity function,  $x_p(\tau, \nu)$  is also delay limited to an interval  $(-T', T')$ , as seen from equation (4), so if  $T' \leq T/2 (\delta \leq 0.5)$  no overlap of the shifted  $x_p$  functions occurs. For  $T' > T/2 (\delta > 0.5)$  the shifted  $x_p$  functions overlap in delay and also in doppler. Because  $x_p(\tau, \nu)$  is extended in doppler (along the delay/doppler ridge) for a chirp signal, overlap between  $x_p$  functions of different orders can take place in the near doppler sidelobes. In particular, sidelobes of  $x_p(\tau \pm T, \nu \pm F)$  may occur along the zero doppler axis and partly inhibit the suppression of grating lobes by  $x_p(\tau, \nu)$ . For the  $\delta = 0.9$  case we have computed, note however that the results are exact for the region in which they are computed,  $|\tau| < 0.1 T$ .

To summarize, the method of computation used in this paper obtains the ambiguity function exactly for  $\delta \leq 0.5$ . When larger duty factors are considered,  $0.5 < \delta < 1.0$ , the method is approximate. For monotone sub-pulses, the degree of approximation is very good, provided  $FT \gg 1$ . For chirp sub-pulses, the approximation is rougher and, if large duty factors are to be seriously considered in an actual application, a more exact calculation, using equation (A-24) instead of equation (1), should be made.

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14. ABSTRACT <p>Linear frequency-stepped pulse trains, periodic in both time and frequency, have ambiguity functions which are periodic in both delay and doppler. One way of suppressing these ambiguities or grating lobes is time and frequency staggering of sub-pulses in the train, smearing out the grating lobes into a delay/doppler residue level. However this works well only with a large number of sub-pulses and a small pulse train duty factor. Another approach, described here, is to chirp code each sub-pulse, filling in the pulse train signal in the frequency domain. This suppresses ambiguities in a strip parallel to the delay axis, giving an unambiguous time response with some doppler tolerance. The technique appears to produce acceptable time responses with either a small or large number of sub-pulses and with various pulse train duty factors. The doppler tolerance, however, depends strongly upon the duty factor.</p>			

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